Linear Transformations

1. Suppose $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}5\\3\\-1\end{bmatrix} \text{ and } T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}2\\1\\1\end{bmatrix}.$$

   (a) What is $T\left(\begin{bmatrix}2\\3\end{bmatrix}\right)$?

   (b) What is the standard matrix of T (i.e. $[T]_{\text{std}}$)?

2. Write the standard matrix for each of the following linear transformations from $\mathbb{R}^2 \to \mathbb{R}^2$.

   (a) Reflection across the line $x_2 = x_1$.

   (b) Rotation by 90° followed by expansion by 3 in the horizontal direction.

   (c) Everything is sent to $\mathbf{0}$.

3. For each matrix below, make a drawing for the function from $\mathbb{R}^2 \to \mathbb{R}^2$ that it defines.

   (a) $\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$ (e) $\begin{bmatrix}0 & -2\\1 & 0\end{bmatrix}$

   (b) $\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$ (d) $\begin{bmatrix}0 & -1\\2 & 0\end{bmatrix}$ (f) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

4. Is the function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}xy\\y\\x\end{bmatrix}$$

   a linear transformation?