Review

For which values of h is the following system consistent?

$$x_1 - hx_2 = 1$$
$$x_1 - x_2 = 0$$

$$\left[\begin{array}{cc|c} 1 & -h & 1 \\ 1 & -1 & 0 \end{array}\right] \xrightarrow{R_2 = R_2 - R_1} \left[\begin{array}{cc|c} 1 & -h & 1 \\ 0 & h-1 & -1 \end{array}\right] \text{ REF}$$

pivot

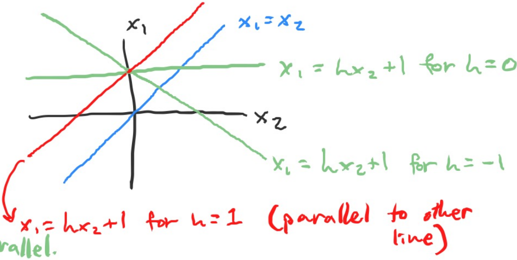
pivot?

Consistent $\Leftrightarrow h - 1 \neq 0 \Leftrightarrow h \neq 1$

Consistent for all values of h other than 1

Geometric meaning?

lines: $x_1 = hx_2 + 1$
$x_1 = x_2$



The first eq'n describes a family of lines, parametrized by h, all with x_1-intercept 1 & slope h.
Always intersect 2nd line except when parallel.

Geometry of Solution Sets

① A is a 3×3 matrix. $\vec{a}, \vec{b}, \vec{c}, \vec{d} \in \mathbb{R}^3$

set of sol'ns to $A\vec{x} = \vec{0}$ is $\text{span}\{\vec{a}, \vec{b}\}$

$A\vec{c} = \vec{d}$

Find all solutions to $A\vec{x} = \vec{d}$

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix} \quad \vec{c} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad \vec{d} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$

Solutions to $A\vec{x} = \vec{d}$ = (one solution to $A\vec{x} = \vec{d}$) + (any solution to $A\vec{x} = \vec{0}$)

$$= \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + s \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \cdot \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$$

for all $s, t \in \mathbb{R}$

② A is 3×4 matrix such that

$$A \xrightarrow[\text{operations}]{\text{row}} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Final answer:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + t \cdot \begin{bmatrix} -5 \\ -2 \\ 0 \\ 1 \end{bmatrix} + s \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

for any $t, s \in \mathbb{R}$

Find all solutions to $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Solutions to $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ = (one solution to $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$)

+ (any solution to $A\vec{x} = \vec{0}$)

Key pt: $\left[\begin{array}{c|c} A & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{array}\right] \rightsquigarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ (row operations do not change all 0's column)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$x_1 = -5\cancel{x_4}\, t$

$x_2 = -2\cancel{x_4}\, t$

x_3 ~~free~~ $= s$

x_4 ~~free~~ $= t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5t \\ -2t \\ s \\ t \end{bmatrix} = t \cdot \begin{bmatrix} -5 \\ -2 \\ 0 \\ 1 \end{bmatrix} + s \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$