Matrices

1. For each of the following, either calculate the product of the matrix and the vector or state that the product is not defined.

(a) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Linear Independence

1. Prove that each of the following lists of vectors is linearly dependent.

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 17 \\ -3 \end{bmatrix}$

(c) $\mathbf{u}, \mathbf{v}, 3\mathbf{u} - 4\mathbf{v}$ where $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^4$.

2. For each list of vectors below, say whether it is linearly dependent or linearly independent.

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

3. Give an example of:

(a) A list of vectors in $\mathbb{R}^2$ which are linearly dependent and span all of $\mathbb{R}^2$.

(b) A list of vectors in $\mathbb{R}^3$ which are linearly independent but do not span all of $\mathbb{R}^3$.