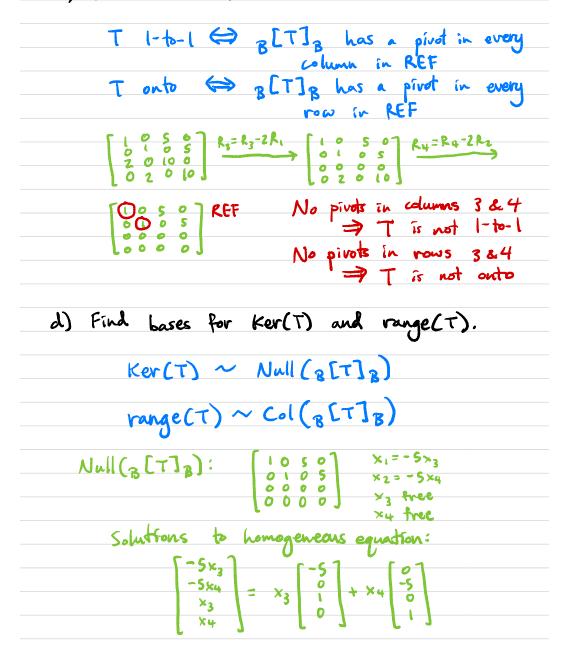


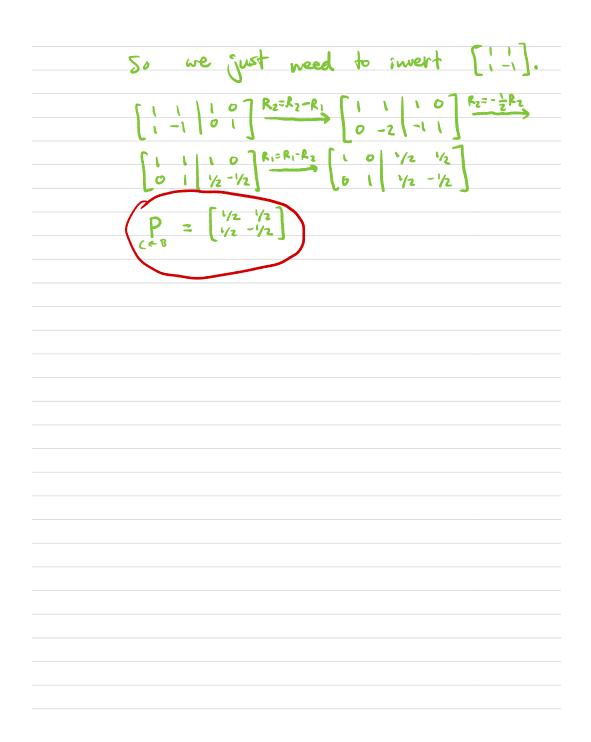
c) Is T one-to-one? Onto?

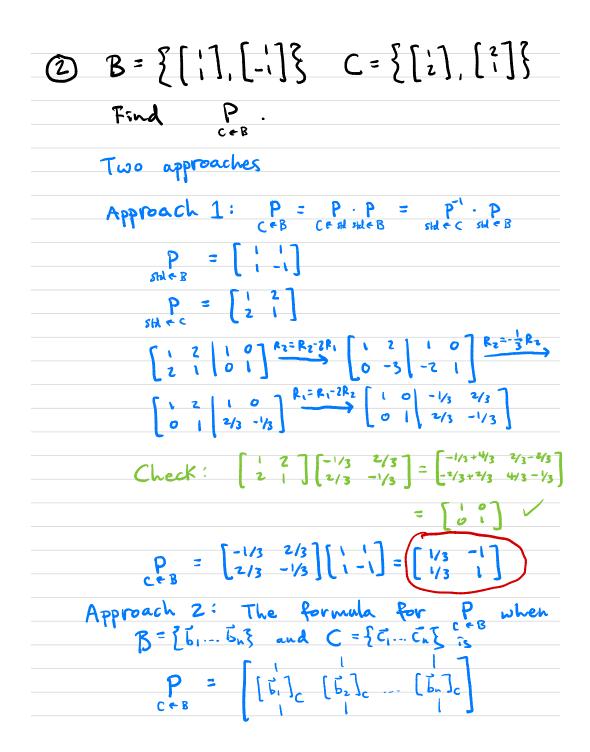


Null (B[T]B) = span { [-5] { -5] } To find a basis for ker(T), translate these vectors back to Mzxz. ker(T) = span $C((B[T]_B): [200]) \longrightarrow [200] (200)$ pivot col $\Rightarrow Col(B[T]_{B}) = span \left\{ \begin{bmatrix} i \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} i \\ 2 \\ 2 \end{bmatrix} \right\}$ To find a basis for range(T), transle these vectors back to MZ+Z $(T) = \operatorname{span} \left\{ \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \right\}$

hange of Basis (] Suppose V is a 2-dimensional vector space and B= { v, v₂ }, C = { u, u₂ } are bases for V such that $\vec{u}_1 = \vec{v}_1 + \vec{v}_2$ a) If $\vec{\omega} \in V$ such that $[\vec{\omega}]_c = \begin{bmatrix} 2\\3 \end{bmatrix}$, what is [w]z? $\begin{bmatrix} \vec{\omega} \end{bmatrix}_{c} = \begin{bmatrix} 2\\ 3 \end{bmatrix} \implies \vec{\omega} = Z \cdot \vec{u}_{1} + 3 \cdot \vec{u}_{2}$ $= 2(\vec{v}_1 + \vec{v}_2) + 3(\vec{v}_1 - \vec{v}_2)$ $= 5\vec{v}_1 - \vec{v}_2$ $S_{o}\left(\left[\vec{\omega}\right]_{B}=\left[\begin{matrix} s\\ -1 \end{matrix}\right]\right)$ b) Find a matrix A such that for all $\vec{x} \in V$, $A[\vec{x}]_c = [\vec{x}]_B$. Suppose [x]c=[x'z]. Then $\vec{x} = x_1 \cdot \vec{u}_1 + x_2 \cdot \vec{u}_2$ $= \times_{1}(\vec{v}_{1} + \vec{v}_{2}) + \times_{2}(\vec{v}_{1} - \vec{v}_{2})$ $= (x_1 + x_2)\vec{v}_1 + (x_1 - x_2)\vec{v}_2$ Hence $[x]_B = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$ This tells us that $A[x_2] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\begin{bmatrix} x_1 + x_2 \\ x_2 - x_2 \end{bmatrix}$ 1^{st} col, of $A = A[b] = [b] \Rightarrow A = [c]$ 2^{st} col of A = A[b] = [b]

The matrix A is the change-ofcoordinates matrix from C to B Caka "change of basis matrix"), written P c) Find a matrix D s.t. for all X eV, $D[\vec{x}]_{B} = [\vec{x}]_{C}.$ In other words, Find P. Two approaches. Approach 1: Guess-and-check We can repeat the method of the solution to part (b) if we can write i, and iz as linear combinations of it, and is. Notice $\vec{u}_1 + \vec{u}_2 = (\vec{v}_1 + \vec{v}_2) + (\vec{v}_1 - \vec{v}_2) = 2\vec{v}_1$ $\vec{u}_1 - \vec{u}_2 = (\vec{v}_1 + \vec{v}_2) - (\vec{v}_1 - \vec{v}_2) = 2\vec{v}_2$ Hence $\vec{v}_i = \frac{1}{2}\vec{u}_i + \frac{1}{2}\vec{u}_s$ ジュニシューシェ 50 $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ Approach Z: Systematic approach Notice that $P = P^{-1}$





So to find CPB we need to find coordinate vectors in the basis C for 1] and I-IT I.e. we need to write [i] and [-i] as linear combinations of [2] and [2]. I.e. we need to find a,b, c, d E R s.t. $\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\} + \left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\} \longrightarrow \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\} \right\}$ $\begin{bmatrix} -1 \\ -1 \end{bmatrix} = c \begin{bmatrix} 2 \\ 2 \end{bmatrix} + d \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\xrightarrow{\mathbf{R}_{2}=-\frac{\mathbf{L}_{2}}{3}} \left[\begin{array}{c} 1 & 2 & 1 & 1 \\ 0 & 1 & \frac{1}{3} & 1 \end{array} \right] \xrightarrow{\mathbf{R}_{1}=\mathbf{R}_{1}-2\mathbf{R}_{2}} \left[\begin{array}{c} 1 & 0 & \frac{1}{3} & -1 \\ 0 & 1 & \frac{1}{3} & 1 \end{array} \right]$ In general if B={b,...bn}, C={c,...cn} are bases for R°, you can find cha by row reducing [imentin by man In C+8]