

MATH 54, FALL 2016, QUIZ 4

(1) Mark each statement true or false. If true, provide a short explanation. If false, provide a counterexample.

(a) A linear transformation from \mathbb{R}^2 to \mathbb{R}^3 cannot be one-to-one.

(b) A linear transformation from \mathbb{R}^2 to \mathbb{R}^3 cannot be onto.

(c) If T and S are both linear transformations from \mathbb{R}^n to \mathbb{R}^m and $T(e_i) = S(e_i)$ for all $i \leq n$ then $T = S$. (Recall that e_i is the vector in \mathbb{R}^n all of whose entries are 0, except for the i^{th} entry, which is 1.)

(d) If a linear transformation is onto then it is also one-to-one.

(2) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation defined by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_2 - x_3 \\ 2x_2 + 4x_3 \end{bmatrix}$$

(a) Find the standard matrix for T .

(b) Is T one-to-one? Is T onto?