Math 10B, Quiz 5

1. (12 points) Suppose you draw 10 cards from a standard deck of 52 cards. What is the probability that you get no hearts or no diamonds? Make sure to clearly explain your answer.

Solution: Let A denote the event that you don't get any hearts and let B be the event that you don't get any diamonds. We want to find $P(A \cup B)$.

By inclusion-exclusion,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

So we need to find $P(A)$, $P(B)$, and $P(A \cap B)$.

Let Ω denote the probability space, which in this case we can take to be the set of all unordered sets of 10 distinct cards from a standard deck. Since each outcome in Ω is equally likely, we can calculate probabilities by finding the size of the different events and the size of Ω:

- $|\Omega| = \binom{52}{10}$ since that's the number of ways to choose ten things from a set of 52 when order doesn't matter.
- $|A| = \binom{39}{10}$ since choosing a set of 10 cards with no hearts is equivalent to choosing ten cards from the 39 cards in the deck that are not hearts.
- $|B| = \binom{39}{10}$ for the same reasons as above.
- $|A \cap B| = \binom{26}{10}$ since $A \cap B$ consists of ways to draw 10 cards so that you don't get hearts and you don't get diamonds and there are 26 cards that are neither hearts nor diamonds.

Therefore the final answer is

$$\frac{\binom{39}{10}}{\binom{52}{10}} + \frac{\binom{39}{10}}{\binom{52}{10}} - \frac{\binom{26}{10}}{\binom{52}{10}}.$$

Common Mistakes: Some people misinterpreted this question as asking about the probability that you get no hearts *and* no diamonds.

Some people didn't realize that adding together the number of ways to get no hearts and the number of ways to get no diamonds double counts the number of ways to get no hearts and no diamonds.

Several people tried to find various probabilities involved in the problem by first finding the probability of the complement, but did not correctly determine what the complement was. For instance, the complement of getting no hearts is getting at least one heart, not getting all hearts.

2. (1 point) Suppose you randomly select a patient at a hospital. Consider the following two events: (1) the patient has lung cancer and (2) the patient has lung cancer and is a smoker. True or false: it is possible that the second event is **more likely** than the first.

○ True √ **False**

Solution: If the patient has lung cancer and is a smoker then it is always true that the patient has lung cancer. In other words, event (2) is a subset of event (1) so it cannot be more likely.

3. (1 point) Suppose that of all the people who died in the US in 2017, 15% died of heart disease, 20% were life–long smokers, and the chance of dying of heart disease for life–long smokers was 40%. True or false: the probability that someone who died of heart disease in the US in 2017 was a life–long smoker was $\frac{.4 \cdot .2}{.15}$.

√ **True** ◯ False

Solution: Let H be the event that a randomly selected person who died in the US in 2017 died of heart disease and let S be the event that they were a life–long smoker. Then the problem is asking about $P(S \mid H)$ and we are told that $P(H) = 0.15$, $P(S) = 0.2$, and $P(H \mid S) = 0.4$. By Bayes' rule $P(S \mid H) = \frac{P(H|S)P(S)}{P(H)}$.

4. (1 point) On an exam, a question asks: "You draw four cards from a standard deck of 52. What is the probability of getting one of each suit." One student gives the answer of $\frac{13^4}{52 \cdot 51 \cdot 50 \cdot 49}$ reasoning as follows: the size of the probability space is $52 \cdot 51 \cdot 50 \cdot 49$ because that is the number of ways to draw four cards from 52 in order. To get one of each suit, you just need to choose which heart, which diamond, which spade, and which club you get and there are 13 options for each of these choices. The student's answer is:

   ◯ Too large

   ◯ Correct

   √ **Too small**

Solution: The student's calculation of the size of the probability space takes the order of cards into account, but the student's calculation of the size of the event does not. In other words, 13^4 is the number of ways to choose one card of each suit, but it does not take into account the order the four cards can be put into. So the student's answer is too small.

Comment: When solving this type of problem, it is often okay to assume that order matters and to assume that order matters, but you need to be consistent (unlike the student in the problem).