Math 10B, Quiz 2

Instructions: For the first question, make sure to show your work and explain your answer. For the final three questions, you do not need to show any work, but you will receive -1 points for an incorrect answer.

1. (12 points) How many ways are there to arrange the letters a, b, c, d, e, and f such that a is not directly followed by either b or c? (For example 'abdefc' and 'acdefb' are both not valid, but 'adbcef' is valid.)

Solution: This is very similar to a homework problem (but with 6 letters instead of just 5 and the extra restriction that c is also not allowed to directly follow a).

We will first count the number of ways to arrange the six letters so that a is followed directly by either b or c.

To find the number of arrangements where a is directly followed by b, we can consider ab as a single letter. Thus there are 5! such arrangements (since we can just put the 5 letters ab, c, d, e, f in any order). And by the same reasoning, the number of arrangements where a is directly followed by c is the same: 5!.

Since it is impossible for a to be *directly* followed by b and c at the same time, there are $2 \cdot 5!$ arrangements where a is directly followed by either b or c.

Since there are 6! total ways to arrange the six letters when there are no restrictions, the answer to the original question is $\boxed{6! - 2 \cdot 5!}$.

Comment: The above solution is not the only valid way to solve this problem, and a number of people successfully used a different approach.

2. (1 point) When using inclusion-exclusion to find the size of the union of 5 sets, you need to subtract the size of the triple intersections.

○ True √ **False**

Solution: The number of sets is actually irrelevant here. When using inclusion-exclusion to find the size of the union of sets $A_1, A_2, \dots, A_n$, you first sum the sizes of all the sets individually then subtract the sizes of all the double intersections, then add the sizes of all the triple intersections and so on, alternating the sign each time.

3. (1 point) The number of nine digit numbers that start with the digit 3 is greater than the number of nine digit numbers in which no digit is repeated.

√ **True** ○ False

Solution: There are 10^8 nine digit numbers that start with 3 (one choice for the first digit and 10 choices for all other digits) and $9 \cdot 9 \cdot 8 \cdot 7 \cdots 3 \cdot 2$ nine digit numbers in which no digits are repeated (nine choices for the first digit since it can't start with 0, nine choices for the second digit since you can't repeat the first digit, and so on). It is straightforward to confirm that the first product is larger.

4. (1 point) On an exam, a question asks "How many strings of 5 letters are there that contain the letter 'z' exactly three times?" One student gives the answer $1 \cdot 1 \cdot 1 \cdot 25 \cdot 25$, reasoning that the three ones are for the three z's and the two 25's are for the other two spots, each of which can be any letter besides z. The student is:

$\surd$ **Undercounting (i.e. their answer is too small)**

○ Correct

○ Overcounting (i.e. their answer is too large)

Solution: This was very similar to a common mistake on the first homework set. The student's mistake is that they are not accounting for all the ways to place the three z's within the string—the three z's are not required to come at the beginning. The correct answer would be $\binom{5}{3} \cdot 25 \cdot 25$ (first choose the positions for the three z's then choose the other two letters) so the student is undercounting.