

Math 10B, Quiz 10

1. (9 points) Solve the following differential equation

$$\frac{t^2 A' - 100A'}{A} = 1.$$

2. (2 points) Separation of variables can be used to solve $y'' = y' + y$.

True False

3. (2 points) $y(t) = \cos(t) + 5$ is a solution to the differential equation $y'(t) \cos(t) + y'(t)y(t) = -5 \sin(t)$. (Hint: the derivative of $\cos(t)$ is $-\sin(t)$.)

True False

4. (2 points) A student is asked to write a differential equation to model the amount of water in a puddle in the following scenario: "A puddle of water initially contains 50 mL of water. Water evaporates from the puddle at a rate proportional to the amount of water in the puddle. There is also a light rain which adds water to the puddle at a rate of 5 mL per minute." The student writes

$$\frac{dW}{dt} = 5t - kW(t); W(0) = 50$$

where $W(t)$ is the amount of water in the puddle (in mL) after t minutes and k is a constant. The student's reasoning is as follows: the derivative of W is how much water is entering the puddle minus how much water is leaving the puddle. After t minutes, $5t$ mL of water have entered the puddle and water is leaving the puddle through evaporation at a rate that is some constant multiple of the amount of water in the puddle. Also, at time 0 there are 50 mL of water in the puddle. The student's answer is:

- Correct with valid reasoning.
 Correct with invalid reasoning.
 Incorrect.