Discrete Probability Worksheet 2: Conditional Probability

1. Suppose there is a test for checking the presence of skin cancer. When cancer is present, the test is positive 90% of the time and negative the other 10%. When cancer is not present, the test is positive 10% of the time, and negative the other 90%. Furthermore, the probability of having cancer is 1%. If someone receives the test and the result is positive, what is the probability that they have cancer? *Hint:* Use Bayes' theorem.

2. Suppose that there are two slot machines, one of which pays out 10% of the time and the other pays out 20% of the time. Unfortunately, you have no idea which is which. Suppose you randomly choose a machine and put in a quarter. If you don't get a jackpot, what is the chance that you chose the machine that pays out 20% of the time? If you had instead gotten a jackpot, what would be the chance that you chose the one that pays out 20% of the time?

3. Kidney stones is an affliction that comes in two varieties: small stones and large stones. Suppose that there are two treatments for kidney stones: treatment A and treatment B. Suppose that the success probabilities of these two types of treatment are as shown in the following table.

	Treatment A	Treatment B
Small Stones	93%	87%
Large Stones	73%	68%

   Also suppose that a patient with kidney stones is equally likely to have small stones or large stones and that patients with small stones receive treatment A with probability 20% and patients with large stones receive treatment A with probability 80%. All patients who don't receive treatment A receive treatment B.

   Given that a patient receives treatment A, what is the chance that it is successful? Given that a patient receives treatment B, what is the chance that it is successful? Which treatment do you think is better?

   By the way, this is a real example. The general phenomenon is known as "Simpson's paradox."

4. Show that your belief in something should never increase both when some other event occurs and when it doesn't occur. Formally, show that if $P(A \mid B) > P(A)$ then $P(A \mid B^c) < P(A)$. By the way, this may seem like an obvious fact, but it has some surprising implications. For instance, if you believe Bitcoin might be a bubble and if the price of Bitcoin rising increases your belief that Bitcoin was a bubble then the price of Bitcoin falling should *decrease* your belief that Bitcoin was a bubble (and vice-versa).

5. Suppose you are playing a game where someone rolls two fair 6-sided dice. If both rolls are ones, you win a million dollars.
   (a) If you are told that the first roll is a one, what is the chance that you will win?
   (b) If you are told that at least one of the rolls is a one, what is the chance that you will win?