

## Combinatorics Worksheet 6: Review and 12-Fold Way

- Suppose that you have  $n$  employees and need to choose some of them to receive a promotion. In each of the following scenarios, how many ways are there to choose which employees receive a promotion?
  - Suppose you can choose any number of employees to receive a promotion.
  - Exactly 5 employees must receive a promotion.
  - Any number of employees can receive a promotion, but at least one of the employees Alan, Kim, and Cassandra must receive a promotion.
  - Any number of employees can receive a promotion, but at *most* one of the employees Alan, Kim, and Cassandra must receive a promotion.
- What is the coefficient of  $x^6y^7$  in  $(3x^2 - y)^{10}$ ?
- Give a combinatorial proof of the fact that for all  $n$  and all  $k \leq n$ ,

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

- How many ways are there to pay your employees if you have \$1000 and 5 employees? Assume that you are allowed to pay employees nothing and that you don't have to spend all \$1000. Also assume that you must pay employees in dollar amounts (e.g. you cannot pay someone \$4.53).
- How many ways are there to arrange 20 books on a bookcase with 3 shelves? Assume, as in real life, that books are distinguishable and that the order of the books on each shelf matters.
- Challenge Problem:** Find as many interesting patterns as possible in Pascal's triangle.