

Combinatorics Worksheet 5: Combinations and Combinatorial Proofs

1. How many anagrams does “obfuscated” have?
2. (a) How many anagrams does “banana” have?
(b) How many anagrams does “banana” have in which the three ‘a’s are next to each other?
(c) How many anagrams does “banana” have in which the three ‘a’s are next to each other and the two ‘n’s are *not* next to each other?
3. (a) Starting from a pool of n people, how many ways are there to select a committee of k people, one of whom is the president of the committee?
(b) Use your answer to part (a) to prove that

$$k \binom{n}{k} = n \binom{n-1}{k-1}.$$

4. Using any method you like (i.e. any chain of sound reasoning), prove that for any m, n , and k such that $k \leq n$ and $k \leq m$,

$$\binom{m+n}{k} = \sum_{i=0}^k \binom{m}{i} \binom{n}{k-i}.$$