Combinatorics Worksheet 4: Permutations

1. (a) 20 people audition for a play with 10 roles. How many ways are there to choose a cast for the play?
   (b) What if actors are allowed to play more than one role each?
2. How many ways are there to arrange a deck of 52 cards so that for each suit, all cards of that suit are together (there are 4 suits of 13 cards each)?
3. Determine the larger number in each pair below. Feel free to experiment on a calculator.
   (a) The number of permutations of a set of size n or the number of subsets of a set of size n?
   (b) The number of 5-permutations of a set of size n or the number of subsets of a set of size n (where n is very large)?
   (c) The number of 5-permutations of a set of size n or the number of $(n-5)$-permutations of a set of size n (when $n > 10$)?
4. Could you plausibly write down all permutations of a set of 5 elements? What about 10? What about 20? How many years would it take to write permutations of 12 elements?
5. Explain why it is not a good idea in Scrabble to simply try out all possible moves each turn.
6. Each square of a 3×7 grid is colored either red or blue. Show that there must be a rectangle in the grid whose corner squares are all the same color. (Hint: apply pigeonhole principle multiple times.)
7. **Challenge Problem:** Show that for any set of 100 integers, there is some non-empty subset whose sum is a multiple of 100.