

Combinatorics Worksheet 3: Pigeonhole Principle

1. A teacher plans to give a five question true/false quiz to her class (300 students total). If any two students have the exact same answers, she will believe they are cheating. Explain why this plan is guaranteed to lead the teacher to accuse some students of cheating even if everyone is honest (of course there are many reasons that this plan is *likely* to lead the teacher to accuse honest students of cheating, but there is also a mathematical reason that the teacher is *guaranteed* to accuse some students of cheating).
2. Show that there are at least 250 four digit numbers whose digits all sum to the same value.
3. Suppose that at a certain college there are only three majors: math, biology and CS. Also suppose that there are:
 - 85 students total
 - 50 math majors, 30 biology majors, and 40 CS majors
 - 10 double majors in math and biology, 20 in math and CS, and 10 in biology and CS.

How many triple majors are there?

4. Show that there are more than 100 people alive right now who were born in the same minute.
5. Show that at least 19 subsets of $\{1, 2, \dots, 10\}$ have the same sum. For a challenge, try to improve this result as much as possible.
6. **Challenge Problem:** Explain why there must be some files whose zipped versions are larger than the original files. Explain why the zip utility is called a compression algorithm despite this fact.