Final Review Worksheet 1

Final Topics

Below is a list of the main topics and types of problems we saw in this class. Bold means that I think that item is fairly likely to show up on the final exam. I cannot promise that this list is exhaustive. Also keep in mind that I have not seen the final exam and so I cannot be certain what questions will appear on it.

- **Combinatorics problems involving addition, multiplication, permutations, combinations**

- **Boxes and balls: stars and bars, stars and bars when every box has to have a ball, Stirling numbers**

- Pigeonhole principle

- Inclusion-Exclusion

- Discrete probability problems that use combinatorics

- **Definition of conditional probability**

- **Bayes' rule for calculating conditional probability**

- Definition of independent events and independent random variables

- Definition of expected value and variance

- Linearity of expectation

- **Distributions (probability mass function, expected value, variance): Bernoulli, binomial, geometric, hypergeometric, Poisson**

- Central limit theorem and law of large numbers

- **Finding 95% confidence intervals, including estimating mean and standard deviation for random variables following Bernoulli, binomial, Poisson, etc. distributions**

- **Hypothesis testing: χ^2 tests for goodness of fit and χ^2 tests for independence. How to perform them, how to interpret the results.**

- **How to solve recurrence relations, with and without initial conditions**

- **How to solve ordinary differential equations: separation of variables, integrating factor, and characteristic polynomial**

- Linear differential equations: definition, how to check if something is linear, and why it is a useful property.

- **Partial fractions for finding some integrals**

- Given some scenario (often involving a tank of salt water for some reason), write a differential equation to model the situation.

- **Euler's method**

- Slope fields

- How to add and multiply matrices, transpose of a matrix, multiplying matrices by scalars and vectors.

- Finding the determinant of a matrix

- **Solving systems of linear equations using Gaussian elimination**

- **Finding the inverse of a matrix using Gaussian elimination**

- **Finding eigenvalues and eigenvectors of a matrix**

- **Finding solutions to systems of linear ODEs (with or without initial conditions) using eigenvalues and eigenvectors**

- Least squares method for linear regression

Review Problems (Combinatorics and Probability)

1. Suppose the final exam is worth 21 points and has six questions.

   (a) How many ways are there to allocate the points? Every question must be worth a positive integer number of points.

   (b) How many ways are there to allocate the points if, in addition to the restrictions in part (a), no question can be worth more than half the total number of points?

2. Suppose you have 20 dogs and 5 cats. You want to put them in a line, but you must have at least 2 dogs between any two cats. How many ways are there to do this? (The dogs and cats are distinguishable.)

3. Suppose you draw 10 cards from a standard 52-card deck.

   (a) What is the probability that you get 10 different suits?

   (b) What is the probability that you get 6 cards of one suit, and 2 cards each of two other suits?

   (c) What is the probability that you get a spade given that you didn't get any hearts?

4. Suppose you have 3 fair dice with 3, 4, and 6 sides (and whose sides are numbered 1–3, 1–4, and 1–6, respectively). You randomly pick one of the dice and roll it.

   (a) Let the random variable X be the number that you rolled. What is $E[X]$?

   (b) If you roll a number less than 4, what is the probability that you picked the 6-sided die?