

Matrix Algebra Worksheet 2

1. Find the inverse of the following matrix

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -5 \\ 0 & 0 & 6 \end{bmatrix}$$

2. (a) Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 6 & 7 \end{bmatrix}$$

- (b) Let A be the matrix from the previous question and suppose that $BC = A$ where B and C are both 3×3 matrices and B is as shown below. Find C^{-1} .

$$B = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

3. True or false:

- (a) The following vector is an eigenvector of the following matrix.

$$\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

- (b) If v is an eigenvector of A then v is also an eigenvector of $5A$.
 (c) If v is an eigenvector of A and of B then it is also an eigenvector of AB .
 (d) If v is an eigenvector of an invertible matrix A then it is also an eigenvector of A^{-1} .
 (e) If v is an eigenvector of A then v is also an eigenvector of A^5 .
4. Find the eigenvalues and eigenvectors of the following matrices.

- (a)

$$\begin{bmatrix} 2 & 1 \\ -2 & 5 \end{bmatrix}$$

- (b)

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

- (c)

$$\begin{bmatrix} 1/2 & -3/5 \\ 3/4 & 11/10 \end{bmatrix}$$

5. **Challenge Question:** When does a 2×2 matrix whose entries are all integers have an inverse whose entries are all integers? What about for an $n \times n$ matrix?
6. **Challenge Question:** Let $p(x)$ be a degree n polynomial. Can you always find an $n \times n$ matrix whose characteristic polynomial is p ?