

## Combinatorics Worksheet 2: Inclusion-Exclusion

1. How many numbers below 100 are divisible by 2,3, or 5?
2. (a) Out of a class of 20 students, how many ways are there to form a study group? Assume that a study group must have at least 2 students.  
(b) How many ways are there to form a study group that contains at least one of Bob, Sue, and Alicia?
3. Suppose you need to come up with a password that uses only the letters A, B, and C and which must use each letter at least once. How many such passwords of length 8 are there?
4. (a) Suppose that 4 people are standing in line. How many ways are there to rearrange the line so that nobody is standing in their original place?  
(b) **Challenge Problem:** Suppose  $n$  people are standing in line and are then randomly put into a new order. When  $n$  is very large, approximately what is the probability that nobody is standing in their original place? (Occasionally I will put challenge problems on the worksheets. I will not go over them in class or post solutions for them, but I encourage you to try to solve them and to ask me about them in office hours.)
5. (a) How many functions from  $\{1, 2, 3, 4, 5\}$  to  $\{A, B, C\}$  are there?  
(b) How many functions from  $\{1, 2, 3, 4, 5\}$  to  $\{A, B, C\}$  are onto?