Dynamics Worksheet 5 Solutions

Find a solution to each of the following differential equations.

1. $A' = 5A$ with initial value $A(0) = -3$.

> **Solution:** Integrate both sides to get the general solution:
> $$A(t) = Ce^{5t}$$
> where C is any constant. Then use the initial value to find C. The final answer is
> $$\boxed{A(t) = -3e^{5t}}$$

2. $y' = 5yx$

> **Solution:** This can be solved using separation of variables or an integrating factor. The solution is
> $$y(t) = Ce^{5x^2/2}$$
> where C is any constant.

3. $y' = 1 + y$

> **Solution:** This can be solved using separation of variables or an integrating factor. The solution is
> $$y(t) = Ce^{x} - 1$$
> where C is any constant.

4. $y' = \frac{x}{x^2y^2 - 4y^2}$ with initial value $y(0) = 1$.

> **Solution:** This can be solved using separation of variables (but not with an integrating factor). The solution is
> $$y(t) = \left(\frac{3}{2}\ln(x^2 - 4) + C\right)^{1/3}$$
> where C is any constant.

5. $\frac{dM}{dt} = 2M(1 - M/5)$

Solution: This can be solved using separation of variables (but not with an integrating factor). The solution is

$$M = \frac{5e^{2x}}{C + e^{2x}}$$

where C is any constant.

6. $\frac{dR}{dx} = \frac{2R+3x^4}{x}$

Solution: This can be solved using an integrating factor (but not with separation of variables). The solution is

$$R(x) = Cx^2 + \frac{3x^4}{2}$$

where C is any constant.

7. $y'' - 2y' - 3y = 0$

Solution: This is a linear, constant coefficient, homogeneous differential equation so we can look at the characteristic polynomial. The roots are 3 and -1, so the general solution is

$$y(t) = C_1 e^{3t} + C_2 e^{-t}$$

where C_1 and C_2 are any constants.

8. $y'' - 6y' + 10y = 0$

Solution: This is a linear, constant coefficient, homogeneous differential equation so we can look at the characteristic polynomial. The roots are $3 + i$ and $3 - i$, so the general solution is

$$y(t) = C_1 e^{(3+i)t} + C_2 e^{(3-i)t}$$

where C_1 and C_2 are any (possibly complex) constants.

If you prefer solutions that don't include complex numbers anywhere then the general solution can also be expressed as

$$y(t) = D_1 e^{3t} \cos(t) + D_2 e^{3t} \sin(t)$$

where D_1 and D_2 are any constants.