

Discrete Probability and Review Worksheet 5

1. How many ways are there to choose a password of length 20 that contains only lowercase letters or digits and that has exactly three a's and at least one b?
2. What is the sum of the coefficients of $(x_1 + x_2 + \dots + x_k)^n$?
3. Suppose we choose a subset of $\{1, 2, \dots, n\}$ by flipping a fair coin to choose whether each element is in the subset or not. Define a random variable X to be the size of the resulting subset. What is $E[X]$?
4. Suppose you repeatedly roll a fair six-sided die until the *second* time you get a six. Define the random variable X to be the number of times you roll the die. Find a formula for $P(X = k)$.
5. Suppose we roll a fair six-sided die twice. Define three random variables as follows: X is the result of the first roll, Y is the sum of the two rolls, and Z is always -2 .
 - (a) Find the probability mass function for each random variable.
 - (b) Find the expected value of each random variable.
 - (c) Find the variance of each random variable.
6. **Challenge Question:** Suppose you are given a biased coin for which the probability of heads is some unknown constant p . How can you use this coin to simulate flipping a *fair* coin? In your scheme, how many times on average do you need to flip the biased coin to simulate one flip of a fair coin?