Combinatorics Worksheet 1

1. How many 4 digit numbers are there which contain only even digits?

2. Suppose you have 10 colors of paint. You can also combine these colors to form new colors. Each combination of the ten original colors gives a different color of paint, but proportions don't matter (so a little green mixed with a lot of red gives the same color as a little red mixed with a lot of green). How many colors can you create, including the original ten colors? (By the way, for the purposes of this problem the absence of paint does not count as a color.)

3. (a) How many possible answer keys are there for a 30 question true/false test?
    (b) How many possible ways are there for a student to fill out a 30 question true/false test? (The student does not need to answer every question.)

4. Suppose you own a small shop that sells t-shirts. There are

    - 6 sizes
    - 5 colors
    - 20 designs
    - 3 types of fabric
    - 6 languages
    - 2 styles (long or short sleeved).

    Is it reasonable for you to stock 10 of each possible type of t-shirt in your store?

5. Suppose that it takes one second to try a password on a computer (I realize that this assumption is not realistic). You want to make sure that nobody can brute-force the password—i.e. determine what the password is simply by sequentially trying all possibilities. In each of the following cases, how long must the password be to make this infeasible? Assume that anything that takes more than a year is not feasible.

   (a) The password may contain only digits.
   (b) The password may contain only uppercase letters.
   (c) The password may contain only digits and uppercase letters.
   (d) The password may contain either only digits or only uppercase letters, but may not contain both digits and uppercase letters.