Math 10B Midterm 1 Review

1. Suppose that you have n employees and need to choose some of them to receive a promotion. In each of the following scenarios, how many ways are there to choose which employees receive a promotion?

   (a) Suppose you can choose any number of employees to receive a promotion.

   (b) Exactly 5 employees must receive a promotion.

   (c) Any number of employees can receive a promotion, but at least one of the employees Alan, Kim, and Cassandra must receive a promotion.

   (d) Any number of employees can receive a promotion, but at *most* one of the employees Alan, Kim, and Cassandra must receive a promotion.

2. Suppose there are 12 people in a room. Show that you can choose two groups of people in the room such that the sum of ages (in years) in both groups is the same.

3. Let x be any positive real number. Show that for all $n \geq 2$, $(1+x)^n > 1 + nx$.

4. How many anagrams does the word "ouroboros" have?

5. Suppose you roll a fair 4-sided die 7 times in a row. What is the probability that all 4 numbers are rolled at least once?

6. What is the coefficient of x^6y^7 in $(3x^2 - y)^{10}$?

7. Suppose you and three of your friends find 100 identical marbles on the ground.

   (a) How many ways are there to divide the marbles between you and your friends?

   (b) How many ways are there to divide the marbles if everybody has to get at least three marbles?

   (c) How many ways are there to divide the marbles if nobody can get more than 30 marbles?