# MATH 10B, SPRING 2017, QUIZ 8

(1) Suppose you roll a die 100 times and 30 of the rolls are fives. Find a 95% confidence interval for the probability $p$ of rolling a five.

If $\widehat{\mu}$ is our estimate for the mean, $\widehat{\text{Var}}$ is our estimate for the variance, and $n$ is the number of trials then the formula for the 95% confidence interval is

$$\left(\widehat{\mu} - 2\sqrt{\frac{\widehat{\text{Var}}}{n}}, \widehat{\mu} + 2\sqrt{\frac{\widehat{\text{Var}}}{n}}\right).$$

We are trying to estimate the mean of a Bernoulli random variable, so if $\widehat{p}$ is our estimate for the mean then our estimate for the variance is $\widehat{p}(1-\widehat{p})$. And based on the given data, our estimate for the mean is

$$\widehat{p} = \frac{30}{100}.$$

Thus

$$\widehat{\text{Var}} = \frac{30}{100}\left(1 - \frac{30}{100}\right) = \frac{21}{100}.$$

There are 100 trials so the 95% confidence interval is

$$\left(\frac{30}{100} - 2\sqrt{\frac{21}{100^2}}, \frac{30}{100} + 2\sqrt{\frac{21}{100^2}}\right).$$

(2) Suppose you want to check if two random variables, $X$ and $Y$, are independent. Assume that both $X$ and $Y$ both take only the values 0 and 1. You collect some data and compile the following table of observations, which records how many times each possible outcome occurred.

| | X = 0 | X = 1 |
|---|---|---|
| Y = 0 | 300 | 100 |
| Y = 1 | 200 | 400 |

(a) Fill in the following table of expected frequencies (i.e. assuming the null hypothesis that the two variables are independent). You do not need to simplify your answers.

*Date*: April 12, 2017.

| | $X = 0$ | $X = 1$ |
|---|---|---|
| $Y = 0$ | $\frac{500 \cdot 400}{1000}$ | $\frac{500 \cdot 400}{1000}$ |
| $Y = 1$ | $\frac{500 \cdot 600}{1000}$ | $\frac{500 \cdot 600}{1000}$ |

(b) Calculate the $\chi^2$ statistic for the given data. You do not need to simplify your answer.

$$\left(\frac{\left(\frac{500 \cdot 400}{1000} - 300\right)^2}{\frac{500 \cdot 400}{1000}}\right) + \left(\frac{\left(\frac{500 \cdot 400}{1000} - 100\right)^2}{\frac{500 \cdot 400}{1000}}\right) + \left(\frac{\left(\frac{500 \cdot 600}{1000} - 200\right)^2}{\frac{500 \cdot 600}{1000}}\right) + \left(\frac{\left(\frac{500 \cdot 600}{1000} - 400\right)^2}{\frac{500 \cdot 600}{1000}}\right)$$

(c) What is the number of degrees of freedom for the $\chi^2$ test on the given data?

(number of possible values of Y $-$ 1)(number of possible values of X $-$ 1)
$= (2-1)(2-1) = 1.$