MATH 10B, SPRING 2017, QUIZ 5

(1) Suppose you draw 10 cards from a standard deck (52 cards, 13 of each suit). After each draw, you put the card back into the deck and reshuffle (so it is possible to draw the same card more than once).

(a) What is the probability that all 10 cards are diamonds?

The sample space is all sequences of 10 cards, with repeats allowed. Each outcome is equally likely, so we use the uniform distribution to calculate the probabilities. The size of the sample space is 52^{10} since there are 52 possibilities for each card and 10 cards total. By the same reasoning, the number of outcomes in which all cards are diamonds is 13^{10}. So the probability is

$$\frac{13^{10}}{52^{10}} = \left(\frac{1}{4}\right)^{10}.$$

Alternatively, you could note that on each draw there is a $1/4$ probability that you get a diamond. Since you put the cards back after each draw, these events are all independent. Thus the probability that you get a diamond on every draw is

$$\left(\frac{1}{4}\right)^{10}.$$

(b) What is the probability that all 10 cards are different (i.e. you never draw the same card twice)?

In the solution to part (a) we already calculated the size of the sample space. Now we need to count the number of outcomes in which all the cards are different, or in other words, the number of ways to choose 10 distinct cards. This is the number of ways to choose 10 cards from 52 where the order of the cards chosen matters. In class, we called this a permutation and denoted it $P(52, 10)$. So the answer is

$$\frac{P(52, 10)}{52^{10}} = \frac{52!}{42!52^{10}}$$

(2) Suppose that 10% of all emails are spam. The word "money" appears in 40% of all spam emails and 1% of all non-spam emails. If an email contains the word "money," what is the probability that it is spam? You do not need to simplify your answer—i.e. it is fine to write an expression with numbers that you could type into a calculator to find the answer.

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Let S be the event that the email is spam and M the event that it contains the word "money." Then the information given in the problem can be summarized as follows:

$$\begin{aligned} P(S) &= 0.1 \\ P(M \mid S) &= 0.4 \\ P(M \mid S^c) &= 0.01. \end{aligned}$$

We are trying to find $P(S \mid M)$. So we can use Bayes' theorem. This gives us

$$\begin{aligned} P(S \mid M) &= \frac{P(M \mid S)P(S)}{P(M)} \\ &= \frac{P(M \mid S)P(S)}{P(M \mid S)P(S) + P(M \mid S^c)P(S^c)} \\ &= \frac{0.4 \cdot 0.1}{0.4 \cdot 0.1 + 0.01 \cdot 0.9} \\ &= \frac{0.04}{0.049} \\ &= \frac{40}{49}. \end{aligned}$$