MATH 10B, SPRING 2017, QUIZ 3

(1) (a) How many anagrams does the word "Mississippi" have?

Each anagram of "Mississippi" is a word of length 11 that contains the same letters as "Mississippi." So to form an anagram, which can first choose which of the 11 letters will be 's,' then which of the remaining 7 letters will be 'i,' then which of the remaining 3 letters will be 'p' and finally, which of the remaining 1 letters will be 'M.' The number of ways to do this is

$$\binom{11}{4}\binom{7}{4}\binom{3}{2}\binom{1}{1} = \binom{11}{4,4,2,1} = \frac{11!}{4!4!2!1!} = \frac{11!}{4!4!2!}.$$

Note: I consider any of the above ways of expressing the answer acceptable. Also, this is not the only way of coming to this solution. Another idea is as follows: first imagine that the different occurrences of 's' and 'i' and 'p' were all distinguishable. Then there are 11! ways to arrange the letters. But since in reality, different occurrences of 's' (and 'i' and 'p') are not distinguishable, some of these 11! possibilities are actually the same. And since there are 4! ways to arrange the four distinguishable versions of the letter 's' or the letter 'i' (and 2! ways to arrange the two distinguishable versions of 'p') we have counted each anagram exactly 4!4!2! times. Which means the total number of anagrams is $\frac{11!}{4!4!2!}$, just as we found above.

(b) How many anagrams does the word "Mississippi" have in which 's' is not the first letter?

First we will count the number of anagrams in which 's' *is* the first letter. If we make 's' the first letter, then the only choice we make is how to arrange the remaining 10 letters. In other words, the number of anagrams in which 's' comes first is the number of anagrams of "Misissippi," where we have removed an 's' from the original word. By the same ideas as in part (a), there are

$$\binom{10}{4,3,2,1} = \frac{10!}{4!3!2!}$$

such anagrams. Thus the number of anagrams that *don't* start with the letter 's' is

$$\frac{11!}{4!4!2!} - \frac{10!}{4!3!2!}.$$

(2) How many ways are there to arrange a deck of 52 cards so that all cards of the same suit are together (there are 4 suits of 13 cards each)?

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To form such an arrangement, we first decide the order of the suits and then the order of cards within each suit. Since there are 4 suits, there are 4! ways to put them in order. For a single suit, there are 13 cards of that suit, and hence 13! ways to put them in order. Since we have to choose an ordering for each of the four different suits, this gives us

$$4!13!13!13!13! = 4!(13!)^4$$

ways to arrange the deck in total.