MATH 10B, SPRING 2017, QUIZ 2

(1) Suppose that at a certain college there are only three majors: math, biology and CS. Also suppose that there are:

- 85 students total
- 50 math majors, 30 biology majors, and 40 CS majors
- 10 double majors in math and biology, 20 in math and CS, and 10 in biology and CS.

How many triple majors are there?

Let M be the set of all math majors, B the set of biology majors and C the set of CS majors. We are trying to find the size of $M \cap B \cap C$. By the principle of inclusion-exclusion

$$\begin{aligned}|M \cup B \cup C| &= |M| + |B| + |C| - |M \cap B| - |M \cap C| - |B \cap C| + |M \cap B \cap C| \\ &= 50 + 30 + 40 - 10 - 20 - 10 + |M \cap B \cap C| \\ &= 80 + |M \cap B \cap C|.\end{aligned}$$

Since $|M \cup B \cup C| = 85$, we have

$$85 = 80 + |M \cap B \cap C|$$

and thus the number of triple majors is $\boxed{5}$.

(2) Show that there are at least 250 four digit numbers whose digits all sum to the same value.

We will use the pigeonhole principle. The boxes will be the possible digit sums of four digit numbers. The objects will be all four digit numbers. Each object will be assigned to a box simply by adding up its four digits. Since 9 is the largest digit, the maximum possible digit sum of a four digit number is

$$9 + 9 + 9 + 9 = 36.$$

On the other hand, every digit sum must be an integer greater than 0. So there are at most 36 boxes.

A four digit number can be formed by choosing on nonzero digit as the first digit and then any digit from 0 to 9 for each of the three remaining digits. So there are

$$9 \cdot 10 \cdot 10 \cdot 10 = 9000$$

four digit numbers. Thus by the pigeonhole principle, some box must have at least

$$\left\lceil \frac{9000}{36} \right\rceil = 250$$

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objects. In other words, there is some integer that is the digit sum of 250 distinct four digit numbers.