Math 10B Probability Review Worksheet

Suppose you repeatedly roll two fair 6-sided dice.

1. What is the probability that the two dice have the same value when you roll both of them once?

   > Let Ω be the set of all pairs of numbers between 1 and 6 (representing the values of the two rolls). Then each outcome in Ω is equally likely. Let A be the event consisting of all pairs of equal values between 1 and 6. Then $|\Omega| = 6^2$ and $|A| = 6$ so
   > $$P(A) = \frac{|A|}{|\Omega|} = \frac{6}{6^2} = \frac{1}{6}.$$

2. What is the probability that both dice have the same value on exactly 5 of the rolls if you roll the dice 10 times each?

   $$\binom{10}{5} \left(\frac{1}{6}\right)^5 \left(1 - \frac{1}{6}\right)^5.$$

3. What is the expected number of times that both dice will have the same value if you roll both 10 times each?

   $$10 \cdot \frac{1}{6}.$$

4. If you keep rolling both dice until you they both have the same value, what is the probability that you roll them 10 times each (i.e. fail 9 times and then succeed)?

   $$\left(1 - \frac{1}{6}\right)^9 \left(\frac{1}{6}\right).$$

5. What is the expected number of times you have to roll before both dice have the same value (i.e. the expected number of failures)?

   $$\frac{1 - \frac{1}{6}}{\frac{1}{6}} = 5.$$

6. Suppose you have a bag of 100 marbles, 10 of which are agates. If you select 15 of the marbles at random, what is the probability that you select 3 agates?

$$\frac{\binom{10}{3}\binom{90}{12}}{\binom{100}{15}}.$$

7. What is the expected number of agates you will select?

$$15 \cdot \frac{10}{100} = \frac{3}{2}.$$