Math 10B Probability Worksheet 2

1. (a) Suppose your friend offers to play the following game with you: she will roll three six-sided dice. If all of the dice show different numbers, she will pay you $10. How much would you be willing to pay her to be allowed to play this game?

   (b) What if she rolls seven dice instead of three?

2. Show that your belief in something should never increase both when some other event occurs and when it doesn't occur. Formally, show that if $P(A \mid B) > P(A)$ then $P(A \mid B^c) < P(A)$.

3. Suppose there is a test for checking the presence of skin cancer. When cancer is present, the test is positive 90% of the time and negative the other 10%. When cancer is not present, the test is positive 10% of the time, and negative the other 90%. Furthermore, the probability of having cancer is 1%. If someone receives the test and the result is positive, what is the probability that they have cancer? *Hint:* Use Bayes' theorem.

4. Kidney stones is an affliction that comes in two varieties: small stones and large stones. Suppose that there are two treatments for kidney stones: treatment A and treatment B. Suppose that the success probabilities of these two types of treatment are as shown in the following table.

	Treatment A	Treatment B
Small Stones	93%	87%
Large Stones	73%	68%

   Also suppose that a patient with kidney stones is equally likely to have small stones or large stones and that patients with small stones receive treatment A with probability 20% and patients with large stones receive treatment A with probability 80%. All patients who don't receive treatment A receive treatment B.

   Given that a patient receives treatment A, what is the chance that it is successful? Given that a patient receives treatment B, what is the chance that it is successful? Which treatment do you think is better?

   By the way, this is a real example. The general phenomenon is known as "Simpson's paradox."