Math 10B Probability Worksheet 1 Solutions

If you are dealt a hand of five cards from a deck of 52, calculate the probability of each of the following events.

1. One pair.

We can find the total number of ways to make a card with exactly one pair as follows. First we choose the value for the pair out of the 13 possible card values. Then we choose the two suits for the pair. Then we choose the values of the remaining three cards from the 12 remaining values (since we should not use the same value as the pair). Then we choose a suit for each of the three singleton cards. This gives us

$$13\binom{4}{2}\binom{12}{3}4^3.$$

On the other hand, the total number of 5 card hands is

$$\binom{52}{5}.$$

So the probability of a hand with exactly one pair is

$$\frac{13\binom{4}{2}\binom{12}{3}4^3}{\binom{52}{5}} \approx 0.4226.$$

2. Two pairs.

$$\frac{\binom{13}{2}\binom{4}{2}^2 44}{\binom{52}{5}} \approx 0.04754$$

3. Three of a kind.

$$\frac{13\binom{4}{3}\binom{12}{2}4^2}{\binom{52}{5}} \approx 0.02113$$

4. Straight—i.e. five cards of sequential value like 8,9,10,J,Q.

The answer here depends on a couple things. One is whether to include straight flushes. The other is whether an ace ranks high, low, or both for the purposes of the straight. We will *not* include straight flushes and we will allow aces to count as either high or low (but not both at once—J, Q, K, A, 2 does not count as a straight but both A, 2, 3, 4, 5 and 10, J, Q, K, A do). We will count the

number of all straights, including straight flushes, and then subtract the number of straight flushes.
There are 10 possible values for the low card in the straight. For any given low card, there are 4^5 possible straights since we can pick the suit of each card in the straight, but the values are all determined by the low card. So there are

$$10 \cdot 4^5$$

total straights. Of these, $10 \cdot 4 = 40$ are straight flushes, since we just need to choose the low card and the suit. Thus there are

$$10 \cdot 4^5 - 40$$

straights that are not flushes, giving a probability of

$$\frac{10 \cdot 4^5 - 40}{\binom{52}{5}} \approx 0.003925.$$

5. Flush—i.e. all cards have the same suit.

We will choose to exclude straight flushes here. As with straights, we will count the total number of flushes and then subtract the number of straight flushes. To form a flush, there are 4 ways to pick the suit and then $\binom{13}{5}$ ways to choose 5 cards of that suit, for a total of

$$4\binom{13}{5}.$$

As above, there are 40 straight flushes, so we have

$$4\binom{13}{5} - 40.$$

flushes that are not straights, giving a probability of

$$\frac{4\binom{13}{5} - 40}{\binom{52}{5}} \approx 0.001965.$$

6. Full house—i.e. three of one value and two of another value.

$$\frac{13\binom{4}{3}12\binom{4}{2}}{\binom{52}{5}} \approx 0.001441$$

7. Four of a kind.

$$\frac{13 \cdot 48}{\binom{52}{5}} \approx 0.0002401$$

8. Straight flush that is not a royal flush.

$$\frac{40 - 4}{\binom{52}{5}} \approx 0.00001385.$$

9. Royal flush—i.e. 10,J,Q,K,A all of one suit.

$$\frac{4}{\binom{52}{5}} \approx 0.000001539$$

10. None of the above.

Add up all the previous answers and subtract them from 1. This yields ≈ 0.5012.