Name: ________________________________

Instructions: This is a mock midterm, designed to give you extra practice for the actual midterm. Good luck!!!

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Date: Monday, September 12th, 2011.
1. *(25 points, 5 pts each)*

Label the following statements as T or F.

Make sure to **JUSTIFY YOUR ANSWERS!!!** You may use any facts from the book or from lecture.

(a) If $A$ and $B$ are square matrices, then $(A + B)^{-1} = A^{-1} + B^{-1}$.

(b) If $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a one-to-one linear transformation, then $T$ is also onto.
(c) If \( \{v_1, v_2, v_3\} \) are linearly independent vectors in \( \mathbb{R}^n \), then \( \{v_1, v_2\} \) is linearly independent as well!

(d) If \( A \) is an invertible square matrix, then \( (A^T)^{-1} = (A^{-1})^T \)
(e) If $A$ is a $3 \times 3$ matrix with two pivot positions, then the equation $Ax = 0$ has a nontrivial solution.
2. (15 points) Solve the following system (or say it has no solutions):

\[
\begin{aligned}
&x + y + z = 0 \\
&2x + 2z = 0 \\
&3x + y + 3z = 0
\end{aligned}
\]
3. (20 points) Find the inverse of the following matrix:

\[
A = \begin{bmatrix}
1 & -1 & 1 \\
1 & 1 & 2 \\
1 & 0 & 1
\end{bmatrix}
\]
4. (10 points) What’s the next elementary row operation you would use to transform the following matrix in row-echelon form? What is the corresponding elementary matrix?

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & -1 \\
0 & 2 & 1
\end{bmatrix}
\]
5. (10 points, 5 points each) Evaluate the following products if they are defined, or say ‘undefined’

(a) $AB$, where:

\[
A = \begin{bmatrix}
2 & 5 \\
0 & 7 \\
-1 & 3
\end{bmatrix},
B = \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]

(b) $AB$, where:

\[
A = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
2 & -1 & 0
\end{bmatrix},
B = \begin{bmatrix}
0 & 1 & 0 \\
2 & 1 & 3 \\
0 & 0 & 1
\end{bmatrix}
\]
6. *(10 points)* Suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation which reflects points in the plane about the origin.

(a) (5 points) Find the matrix $A$ of $T$.

(b) (5 points) Use $A$ to find $T(1,1)$. 
7. (10 points) Find a basis for $\text{Nul}(A)$ and $\text{Col}(A)$, where $A$ is the following matrix:

$$A = \begin{bmatrix}
1 & 1 & 3 \\
0 & -1 & 1 \\
0 & 1 & 2
\end{bmatrix}$$