Name: __________________________________________

Instructions: This midterm counts for 20% of your grade. You officially have 50 minutes to take this exam (although I will try to give you more time). Good luck, and don’t worry, you’ll be fine!

Note: Please check the following box if it applies to you:

☐ I am taking a Summer Session A course (May 21 - June 29), and I feel that this has prevented me from showing you my full math potential

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Date: Friday, June 29th, 2012.
Label the following statements as TRUE (T) or FALSE (F). Write your answers in the box below!

**NOTE:** In this question, you do NOT have to justify any answers! Also, don’t spend too much time on each question!

(a) If the augmented matrix of the system $A\mathbf{x} = \mathbf{b}$ has a pivot in the last column, then the system $A\mathbf{x} = \mathbf{b}$ has no solution.

(b) If $A$ and $B$ are invertible $2 \times 2$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$

(c) If $A$ is a $3 \times 3$ matrix such that the system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^3$.

(d) The general solution to $A\mathbf{x} = \mathbf{b}$ is of the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_0$, where $\mathbf{x}_p$ is a particular solution to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_0$ is the general solution to $A\mathbf{x} = \mathbf{0}$.

(e) If $P$ and $D$ are $n \times n$ matrices, then $\det(PDP^{-1}) = \det(D)$
2. (10 points, 5 points each) Label the following statements as TRUE or FALSE. In this question, you HAVE to justify your answer!!!

This means:

- If the answer is TRUE, you have to explain WHY it is true (possibly by citing a theorem)

- If the answer is FALSE, you have to give a specific COUNTEREXAMPLE. You also have to explain why the counterexample is in fact a counterexample to the statement!

(a) If $A$ and $B$ are any $2 \times 2$ matrices, then $AB = BA$

(b) The matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$ is not invertible.
3. (15 points) Solve the following system of equations (or say it has no solutions):

\[
\begin{align*}
2x + 2y + z &= 2 \\
3x + 4y + 2z &= 3 \\
x + 2y - z &= -3
\end{align*}
\]
4. (20 points) Solve the following system $Ax = b$, where:

$$A = \begin{bmatrix} 1 & 1 & 1 & -3 \\ 2 & 3 & 1 & -6 \\ -1 & 2 & -4 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 8 \\ 3 \end{bmatrix}$$

Write your answer in (parametric) vector form
5. (15 points, 5 points each)
   (a) Calculate $AB$, or say that $AB$ is undefined.

   \[ A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \]

   (b) Calculate $AB$, or say that $AB$ is undefined.

   \[ A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 3 & 0 \end{bmatrix} \]

   (c) Calculate $A^2$, where:

   \[ A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]
6. (15 points) Find $A^{-1}$ (or say ‘$A$ is not invertible’) where:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$
7. (15 points) Find \( \det(A) \), where:

\[
A = \begin{bmatrix}
1 & 0 & 0 & 3 & 1 \\
2 & 0 & 4 & 0 & 5 \\
1 & 2 & 5 & -2 & 0 \\
2 & 0 & 3 & 0 & 1 \\
0 & 0 & 1 & 0 & -1
\end{bmatrix}
\]
**Bonus (3 points)** Find $\text{det}(A)$, where:

\[
A = \begin{bmatrix}
1 & x & x^2 & x^3 \\
1 & y & y^2 & y^3 \\
1 & z & z^2 & z^3 \\
1 & t & t^2 & t^3 \\
\end{bmatrix}
\]

where $x, y, z, t$ are distinct real numbers. This is called a Vandermonde matrix!

**Hint:** Calculating this directly is going to drive you nuts! Can you think about another way of calculating the determinant?

**Note:** You might find the formula $p^3 - q^3 = (p - q)(p^2 + pq + q^2)$ (where $p$ and $q$ are real numbers) useful!
(Scratch work)
Any comments about this exam? (too long? too hard?)