MATH 54 – QUIZ 8 – SOLUTIONS

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1. *(5 points)* Find a basis for the eigenspace E_2 of A corresponding to the eigenvalue $\lambda = 2$, where:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix}$$

All we need to find is a basis of $Nul(2I - A)$.

$$Nul(2I - A) = Nul \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} = Nul \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

But if $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, then $y - z = 0$ and $x = 0$, so $y = z$ and $x = 0$, hence:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ z \\ z \end{bmatrix} = z \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Hence a basis for E_2 is $\boxed{\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}}$

Date: Friday, October 17, 2014.

2. *(5 points)* Find the characteristic polynomial and the eigenvalues of the matrix A, where:

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

The characteristic polynomial is:

$$\begin{aligned} \det(\lambda I - A) = & \begin{vmatrix} \lambda - 2 & -3 \\ -3 & \lambda - 2 \end{vmatrix} \\ = & (\lambda - 2)^2 - 9 \\ = & (\lambda - 2 - 3)(\lambda - 2 + 3) \\ = & (\lambda - 5)(\lambda + 1) \\ = & \lambda^2 - 4\lambda - 5 \end{aligned}$$

To find the eigenvalues, solve $\det(\lambda I - A) = (\lambda - 5)(\lambda + 1) = 0$, which implies $\lambda = 5$ or $\lambda = -1$.

Hence the eigenvalues of A are $\boxed{\lambda = -1, 5}$