MATH 54 – QUIZ 7 – SOLUTIONS

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1. *(5 points)* Consider the following subspace W of $\mathbb{R}^3$, given by all vectors of the following form, where a, b, c, d are real numbers.

$$\begin{bmatrix} a - 3b - 2c - 3d \\ -2a + 6b + 3c + 5d \\ 5c + 5d \end{bmatrix}$$

(a) *(3 points)* Find a basis for W

Notice that we can write W as $Col(A)$, where:

$$A = \begin{bmatrix} 1 & -3 & -2 & -3 \\ -2 & -6 & 3 & 5 \\ 0 & 0 & 5 & 5 \end{bmatrix} \begin{bmatrix} 1 & -3 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Notice that there are pivots in the first and third columns of A, hence a basis for $W = Col(A)$ is:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix} \right\}$$

(b) *(2 points)* Find $dim(W)$

There are two vectors in $\mathcal{B}$, hence $\boxed{dim(W) = 2}$

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Date: Friday, October 10, 2014.

2. *(5 points)* If A is a 5×8 matrix, what is the **largest** possible dimension of $Row(A)$? What is the **smallest** possible dimension of $Nul(A)$?

   A can have at most 5 pivots, hence the largest possible dimension of $Row(A)$ is $\boxed{5}$

   (remember that $dim(Row(A)) = Rank(A) =$ number of pivots).

   By the rank theorem, $dim(Nul(A)) + rank(A) = 8$.

   Therefore $dim(Nul(A)) = 8 - rank(A) \geq 8 - 5 = 3$, so the smallest possible dimension of $Nul(A)$ is $\boxed{3}$