MATH 54 – QUIZ 6 – SOLUTIONS

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1. *(5 points)* Is the set W of 2×2 **antisymmetric** matrices (with real entries) a subspace of the set V of all 2×2 matrices (with real entries)?

(A matrix A is antisymmetric if and only if $A^T = -A$. Equivalently[1], an antisymmetric 2×2 matrix is of the form $\begin{bmatrix} a & -b \\ b & c \end{bmatrix}$, where a, b, c are in $\mathbb{R}$.)

There are two ways of solving this problem, choose your favorite one!

Solution 1: All we need to show that the zero-matrix is in W, and that W is closed under addition and scalar multiplication.

Zero-vector: The 2×2 zero-matrix $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ satisfies $O^T = O = -O$, therefore O is in W

Closed under addition: Suppose A and B are in W. Then $A^T = -A$ and $B^T = -B$. But then, by properties of transpose:

$$(A+B)^T = A^T + B^T = -A - B = -(A+B)$$

And therefore $A + B$ is antisymmetric. Since A and B were arbitrary in W, it follows that W is closed under addition

Closed under scalar multiplication: Suppose A in in W and c is a real number, then $A^T = 0$, and so, by properties of transposes:

$$(cA)^T = c\left(A^T\right) = c(-A) = -(cA)$$

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[1]Technically, those are not the same, but for the purpose of this quiz assume they're the same

And therefore cA is antisymmetric. Since A and c were arbitrary, it follows that W is closed under multiplication.

Therefore, W is a subspace of V.
<u>Solution 2:</u>
Notice that every matrix $\begin{bmatrix} a & b \\ -b & c \end{bmatrix}$ can be written as:

$$\begin{bmatrix} a & b \\ -b & c \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, it follows that:

$$W = Span \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

But since the span of any number of vectors in V is a subspace of V, it follows that W is a subspace of V.

2. *(5 points)* Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \end{bmatrix} \right\}$ be a basis of $\mathbb{R}^2$.

   (a) *(2 points)* Calculate the change-of-coordinates matrix $P_{\mathcal{B}}$ from $\mathcal{B}$ to the standard basis of $\mathbb{R}^2$

$$P_{\mathcal{B}} = \begin{bmatrix} 1 & 3 \\ -2 & -5 \end{bmatrix}$$

   (b) *(3 points)* Use (a) to calculate $[\mathbf{x}]_{\mathcal{B}}$ given $\mathbf{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

   We know that:

$$\mathbf{x} = P_{\mathcal{B}} [\mathbf{x}]_{\mathcal{B}}$$

   And therefore:

$$[\mathbf{x}]_{\mathcal{B}} = (P_{\mathcal{B}})^{-1} \mathbf{x} = \begin{bmatrix} 1 & 3 \\ -2 & -5 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

3. *(0 points)* Assuming that your happiness is a vector space, what is the basis of your happiness?

Anything from Math 54 to chocolate works :)