

MATH 54 – QUIZ 5 – SOLUTIONS

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Name: _____

Instructions: You have 20 minutes to take this quiz, for a total of 10 points. Good luck, and may Bomberman be with you!

1. (5 points) Suppose A is a 7×8 matrix such that when you solve $A\mathbf{x} = \mathbf{0}$, you get exactly one free variable. Do the columns of A span \mathbb{R}^7 ? Explain why or why not, using a theorem learned in this course.

By the rank-nullity theorem, we have:

$$\dim(\text{Nul}(A)) + \dim(\text{Col}(A)) = 8$$

However, the fact that there is just one free variable tells us that $\dim(\text{Nul}(A)) = 1$ (that is, $\text{Nul}(A)$ is spanned by one nonzero vector) hence:

$$\dim(\text{Col}(A)) = 8 - \dim(\text{Nul}(A)) = 8 - 1 = 7$$

Therefore, $\text{Col}(A)$ is a 7-dimensional subspace of \mathbb{R}^7 , and thus $\text{Col}(A) = \mathbb{R}^7$. Hence it follows that the columns of A span \mathbb{R}^7 (by definition of $\text{Col}(A)$).

2. (5 points) Calculate the following determinant:

$$\begin{aligned} \begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix} &= (-3) \begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix} \\ &= (-3) \left(5 \begin{vmatrix} -2 & 2 \\ -6 & 5 \end{vmatrix} - 0 \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix} \right) \\ &= (-3)(5 \times 2 + 4 \times (-2)) \\ &= (-3)(2) \\ &= -6 \end{aligned}$$