MATH 54 – QUIZ 4 – SOLUTIONS

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Instructions: You have 20 minutes to take this quiz, for a total of 10 points. May your luck be invertible!

1. *(5 points)*

   (a) *(2 points)* Use **row-reduction** to find A^{-1}, where: $A = \begin{bmatrix} 1 & -4 \\ 2 & -7 \end{bmatrix}$

   Setting up our giant matrix and row-reducing (here we subtracted 2 times the first row from the second, and added 4 times the second row to the first), we get:

$$\begin{bmatrix} 1 & -4 & 1 & 0 \\ 2 & -7 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -7 & 4 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

   Therefore: $A^{-1} = \begin{bmatrix} -7 & 4 \\ -2 & 1 \end{bmatrix}$

   (b) *(3 points)* Let T and S be linear transformations such that the matrix of T is A and the matrix of S is B, where:

   $A = \begin{bmatrix} 1 & -4 \\ 2 & -7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

   Using (a), find the matrix of $T^{-1} \circ S$.

   The matrix in question is:

$$A^{-1}B = \begin{bmatrix} -7 & 4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 14 \\ 1 & 2 & 4 \end{bmatrix}$$

Date: Friday, September 19th, 2014.

2. *(5 points)* Find a basis for $Col(A)$ and a basis for $Nul(A)$, where A is the following matrix (with the following row-echelon form):

$$A = \begin{bmatrix} 2 & 4 & -5 & 2 & -3 \\ 3 & 6 & -8 & 3 & -5 \\ 0 & 0 & 9 & 0 & 9 \\ -3 & -6 & -7 & -3 & -10 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 2 & -5 & 1 & -4 \\ 0 & 0 & \boxed{5} & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for Col(A): Notice that there are pivots in the first and third column of A, therefore a basis for $Col(A)$ is (it's crucial to go back to the **original** columns of A):

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -5 \\ -8 \\ 9 \\ -7 \end{bmatrix} \right\}$$

Basis for Nul(A): Row-reducing the matrix until we get the RREF (here we added the second row to the first and divided the second row by 5), we get that the RREF of A is:

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now solving $A\mathbf{x} = \mathbf{0}$, we get that $x = -2y - t - s$ and $z = -s$, and therefore:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \\ t \\ s \end{bmatrix} = \begin{bmatrix} -2y - t - s \\ y \\ -s \\ t \\ s \end{bmatrix} = \begin{bmatrix} -2y \\ y \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ 0 \\ t \\ 0 \end{bmatrix} + \begin{bmatrix} -s \\ 0 \\ -s \\ 0 \\ s \end{bmatrix} = y \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Thereofre a basis for $Nul(A)$ is:

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$