MATH 54 – QUIZ 2 – SOLUTIONS

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Name:________________________

Instructions: Greetings from New York! You have 20 minutes to take this quiz, for a total of 10 points. After you hand in your quiz, you're allowed to leave, as the rest of section is cancelled. Good luck, and remember: I've got one less row-reduction, one less row-reduction!

1. *(5 points)* Find the general solution of the linear system corresponding to the following **augmented** matrix.

$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 2 & -1 & 5 & 0 \\ -12 & 3 & -6 & -6 \end{bmatrix}$$

First, divide the third row by -3:

$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 2 & -1 & 5 & 0 \\ 4 & -1 & 2 & 2 \end{bmatrix}$$

Then subtract 2 times the first row from the second, and 4 times the first row from the third:

$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & -7 & 7 & -14 \\ 0 & -13 & 6 & -26 \end{bmatrix}$$

Then divide the second row by -7:

$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & 1 & -1 & 2 \\ 0 & -13 & 6 & -26 \end{bmatrix}$$

Then add 13 times the second row to the third:

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$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

Then you can either backsubstitute, or continue until you reach the reduced row-echelon form. For the latter, divide the third row by -7:

$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

And subtract the third row from the first row, and subtract the third row from the second row:

$$\begin{bmatrix} 1 & 3 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Finally, subtract 3 times the second row from the first:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

And we obtain our solution:

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 0 \end{cases}$$

2. *(5 points)* Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 1 & -2 & -1 \\ 3 & -4 & 11 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ -5 \\ 15 \end{bmatrix}$$

First form the matrix:

$$\begin{bmatrix} 2 & -3 & 5 & 5 \\ 1 & -2 & -1 & -5 \\ 3 & -4 & 11 & 15 \end{bmatrix}$$

And now row-reduce it (what else?). For this, we first interchange the first and second rows (it's better to have 1's on top):

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 2 & -3 & 5 & 5 \\ 3 & -4 & 11 & 15 \end{bmatrix}$$

Now subtract 2 times the first row from the second, and 3 times the first row from the third:

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 0 & 1 & 7 & 15 \\ 0 & 2 & 14 & 30 \end{bmatrix}$$

Now divide the third row by 2:

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 0 & 1 & 7 & 15 \\ 0 & 1 & 7 & 15 \end{bmatrix}$$

And subtract the second row from the third:

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 0 & 1 & 7 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution 1: This is in row-echelon form, with two pivots 1 and 1. Notice that there are no rows of the form $\begin{bmatrix} 0 & 0 & 0 & \star \end{bmatrix}$ where $\star \neq 0$, hence the system is **consistent** (by the fact discussed in section), and therefore $\boxed{\text{b is a linear combination of the columns of } A}$.

Solution 2: You could either say the above, or just continue row-reducing. For this, add 2 times the second row to the first:

$$\begin{bmatrix} 1 & 0 & 13 & 25 \\ 0 & 1 & 7 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which gives $x_1 + 13x_3 = 25$, that is $x_1 = 25 - 13x_3$, and $x_2 + 7x_3 = 15$, that is $x_2 = 15 - 7x_3$.

Therefore, the solution of $A\mathbf{x} = \mathbf{b}$ is:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 25 - 13x_3 \\ 15 - 7x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 15 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -13 \\ -7 \\ 1 \end{bmatrix}$$

Therefore $\mathbf{b}$ is a linear combination of the columns of A, with weights $25 - 13x_3$, $15 - 7x_3$, and x_3, where x_3 is free.