MATH 54 – QUIZ 2 – SOLUTIONS

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Name: ____________________________

Instructions: Greetings from New York! You have 20 minutes to take this quiz, for a total of 10 points. After you hand in your quiz, you’re allowed to leave, as the rest of section is cancelled. Good luck, and remember: I’ve got one less row-reduction, one less row-reduction!

1. (5 points) Find the general solution of the linear system corresponding to the following augmented matrix.

\[
\begin{bmatrix}
1 & 3 & -1 & 7 \\
2 & -1 & 5 & 0 \\
-12 & 3 & -6 & -6
\end{bmatrix}
\]

First, divide the third row by $-3$:

\[
\begin{bmatrix}
1 & 3 & -1 & 7 \\
2 & -1 & 5 & 0 \\
4 & -1 & 2 & 2
\end{bmatrix}
\]

Then subtract 2 times the first row from the second, and 4 times the first row from the third:

\[
\begin{bmatrix}
1 & 3 & -1 & 7 \\
0 & -7 & 7 & -14 \\
0 & -13 & 6 & -26
\end{bmatrix}
\]

Then divide the second row by $-7$:

\[
\begin{bmatrix}
1 & 3 & -1 & 7 \\
0 & 1 & -1 & 2 \\
0 & -13 & 6 & -26
\end{bmatrix}
\]

Then add 13 times the second row to the third:

\[
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\]
Then you can either backsubstitute, or continue until you reach the reduced row-echelon form. For the latter, divide the third row by $-7$:

$$
\begin{bmatrix}
1 & 3 & -1 & 7 \\
0 & 1 & -1 & 2 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

And subtract the third row from the first row, and subtract the third row from the second row:

$$
\begin{bmatrix}
1 & 3 & 0 & 7 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

Finally, subtract $3$ times the second row from the first:

$$
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

And we obtain our solution:

\[
\begin{align*}
x_1 &= 1 \\
x_2 &= 2 \\
x_3 &= 0
\end{align*}
\]

2. (5 points) Determine if $b$ is a linear combination of the vectors formed from the columns of the matrix $A$.

$$
A = \begin{bmatrix}
2 & -3 & 5 \\
1 & 1 & -1 \\
3 & -4 & 11
\end{bmatrix},
$$

$$
b = \begin{bmatrix}
5 \\
-5 \\
15
\end{bmatrix}
$$

First form the matrix:
And now row-reduce it (what else?). For this, we first interchange the first and second rows (it’s better to have 1’s on top):

\[
\begin{bmatrix}
1 & -2 & -1 & -5 \\
2 & -3 & 5 & 5 \\
3 & -4 & 11 & 15
\end{bmatrix}
\]

Now subtract 2 times the first row from the second, and 3 times the first row from the third:

\[
\begin{bmatrix}
1 & -2 & -1 & -5 \\
0 & 1 & 7 & 15 \\
0 & 2 & 14 & 30
\end{bmatrix}
\]

Now divide the third row by 2:

\[
\begin{bmatrix}
1 & -2 & -1 & -5 \\
0 & 1 & 7 & 15 \\
0 & 1 & 7 & 15
\end{bmatrix}
\]

And subtract the second row from the third:

\[
\begin{bmatrix}
1 & -2 & -1 & -5 \\
0 & 1 & 7 & 15 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

**Solution 1:** This is in row-echelon form, with two pivots 1 and 1. Notice that there are no rows of the form \([0 \ 0 \ \star]\) where \(\star \neq 0\), hence the system is **consistent** (by the fact discussed in section), and therefore \(b\) is a linear combination of the columns of \(A\).

**Solution 2:** You could either say the above, or just continue row-reducing. For this, add 2 times the second row to the first:

\[
\begin{bmatrix}
1 & 0 & 13 & 25 \\
0 & 1 & 7 & 15 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
which gives $x_1 + 13x_3 = 25$, that is $x_1 = 25 - 13x_3$, and $x_2 + 7x_3 = 15$, that is $x_2 = 15 - 7x_3$.

Therefore, the solution of $Ax = b$ is:

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 25 - 13x_3 \\ 15 - 7x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 15 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -13 \\ -7 \\ 1 \end{bmatrix}
$$

Therefore $b$ is a linear combination of the columns of $A$, with weights $25 - 13x_3$, $15 - 7x_3$, and $x_3$, where $x_3$ is free.