

## MATH 54 – QUIZ 1 – SOLUTIONS

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Name: \_\_\_\_\_

**Instructions:** You have 20 minutes to take this quiz, for a total of 10 points. Good luck, and don't worry, everything will be fine.

1. (5 points) Assume the following assertions are true:

“There are planets in the universe”

“On every planet, there is an alien who thinks that all the stars are made out of whipped cream”

To make this more understandable, let's call that alien a 'troll,' so this says that on every planet there is a troll, and everyone who is not a troll is 'sane.'

Which of the following assertions must necessarily be true?

(a) There is a planet on which every alien believes no stars are made out of whipped cream.

(**FALSE**, because on every planet there is a troll. So it's not true that every alien is sane)

(b) There are no planets on which every alien believes no stars are made out of whipped cream.

(**TRUE**, because there are no planets on which every alien is sane, because there are trolls)

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- (c) There is no planet on which every alien does not believe some stars are made out of whipped cream.

(**TRUE**, because if a troll believes all stars are made out of whipped cream, then it believes that some stars are made out of whipped cream)

- (d) There is a planet on which there is an alien who believes all stars are made out of whipped cream.

(**TRUE**, because if on every planet there is a troll, then on some planet there is a troll)

- (e) There is a planet on which every alien believes some star is made out of whipped cream.

(**FALSE**, there *could* be a planet full of trolls, but it doesn't *necessarily* have to be a planet full of trolls)

2. (5 points) For which numbers  $a, b, c$  is the following matrix in Row-echelon form or Reduced row-echelon form?

$$\begin{bmatrix} 1 & b & c \\ 0 & a & b \\ 0 & 0 & a \end{bmatrix}$$

**Note:** For this problem, it's best to proceed case-by-case, depending on whether  $a \neq 0$  or not, and then (if necessary)  $b \neq 0$  or not, and  $c \neq 0$  or not (it's best to start with  $a$ , and then do  $b$  and  $c$ )

Row-echelon form:

Case 1:  $a \neq 0$ , then there is a pivot in every row and the matrix is in row-echelon form.

Case 2:  $a = 0$

Case 2a:  $b \neq 0$ , then there is a pivot 1 in the first row,  $b$  in the second row, and the last row is all 0, and the matrix is in row-echelon form

Case 2b:  $b = 0$ , then there is a pivot 1 in the first row, and the second and third rows are all 0.

**Conclusion:** It turns out that in all cases the matrix is in row-echelon form, and hence the answer is  $\boxed{\text{all } a, b, c \in \mathbb{R}}$  (the analysis of  $c$  wasn't even needed)

Reduced row-echelon form:

Case 1:  $a \neq 0$ , and in this case  $a$  is a pivot and so we get  $a = 1$  (this is one of the requirements of the RREF), then there is a pivot 1 in the first column, a pivot  $a = 1$  in the second column, and a pivot  $a = 1$  in the third column. By the second requirement of the RREF, it follows that  $b = 0$  and  $c = 0$  (remember that in the RREF, all the entries in the pivot column must be 0, except for the pivot which must be 1), and this gives the RREF:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Case 2:  $a = 0$

Case 2a:  $b \neq 0$ , and in this case  $b$  is a pivot and so we get  $b = 1$ . Furthermore, there is a pivot 1 in the first column and a pivot  $b = 1$  in the third column, and hence  $c$  must be 0, which gives the RREF:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Case 2b:  $b = 0$ , and in this case there is only one pivot 1 in the first column, which gives the following RREF:

$$\begin{bmatrix} 1 & 0 & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

**Conclusion:** There are 3 possibilities:

$a = 1, b = 0, c = 0$  which gives the first matrix

$a = 0, b = 1, c = 0$ , which gives the second matrix

$a = 0, b = 0, c \in \mathbb{R}$ , which gives the third matrix.