MATH 54 – HINTS TO HOMEWORK 9

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Here are a couple of hints to Homework 9. Enjoy!

SECTION 4.1: VECTOR SPACES AND SUBSPACES

Remember the three techniques of showing whether something is a vector space or not!

(1) Trick 1: Show it is not a vector space by finding an explicit property which does not hold
(2) Trick 2: Show it is a subspace of a (known) vector space
(3) Trick 3: Express it in the form $Span$ of some vectors.

4.1.13(c). For (c), to show $\mathbf{w}$ is in the subspace or not, all you have to show is whether the system $A\mathbf{x} = \mathbf{w}$ is consistent or not (where A is the matrix whose columns are the $\mathbf{v}_i$).

4.1.24.

(a) **T** (this is important to remember!!! A vector isn't a list of numbers any more, it could be anything, even a function!)
(b) **T**
(c) **T** (of itself!)
(d) **F**
(e) **T** (again, the textbook might give you a different answer, but I agree that this is weirdly phrased! What they mean is: If $\mathbf{u}, \mathbf{v}$ is in H, then $\mathbf{u} + \mathbf{v}$ is in H).

4.1.32. This is a bit tricky! Remember that $H \cap K$ is the set of vectors that is both in H and in K. Here's the proof that $H \cap K$ is closed under addition (hopefully that'll inspire you to do the rest):

Suppose $\mathbf{u}$ and $\mathbf{v}$ are in $H \cap K$. Then $\mathbf{u}$ and $\mathbf{v}$ are in H, so is $\mathbf{u} + \mathbf{v}$ (since H is a subspace). Also, since $\mathbf{u}$ and $\mathbf{v}$ are in K, so is $\mathbf{u} + \mathbf{v}$ (since K is a subspace). Hence $\mathbf{u}+\mathbf{v}$ is both in H and K, hence $\mathbf{u}+\mathbf{v}$ is in $H \cap K$.

Date: Wednesday, October 1st, 2014.

As for the fact that the union of two subspaces is not a subspace, take H to be the $x-$axis, and K to be the $y-$ axis. Then $(1,0)$ and $(0,1)$ are both in the union, but $(1,1)$ is not.

Section 4.2: Nullspaces, Column Spaces, and Linear Transformations

This is very similar to what you've been doing in sections 2.8 and 2.9. See also the tricks I gave in the beginning of section 4.1.

4.2.23. Is $\mathbf{w}$ a linear combo of the columns of A? Is $A\mathbf{w} = \mathbf{0}$?

4.2.25.

(a) **T**
(b) **F**
(c) **T**
(d) **T** (the book might say **F**, if it is pedantic about the fact that it didn't say 'for all $\mathbf{b}$')
(e) **T**
(f) **T**