MATH 54 – SOLUTION TO 2.2.13

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Suppose $AB = AC$, where A is invertible. Then, multiplying both sides to the left by A^{-1}, we get:

$$\begin{aligned}
A^{-1}(AB) =& A^{-1}(AC) \\
(A^{-1}A)B =& (A^{-1}A)C && \text{(by associativity)} \\
IB =& IC && \text{(because } A^{-1}A = I\text{, by definition of inverse)} \\
B =& C && \text{(because } ID = I \text{ for every matrix } D\text{, by definition of } I\text{)}
\end{aligned}$$

And therefore $B = C$.

The result is not true in general, because if you take $A = O$ (the zero-matrix), then $AB = OB = O$, and $AC = OC = O$, and hence $AB = AC$, but $B \neq C$.

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