

MATH 54 – SOLUTION TO 1.7.5

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We want to solve the system $Ax = \mathbf{0}$. The augmented matrix becomes:

$$\begin{bmatrix} 0 & -3 & 9 & 0 \\ 2 & 1 & -7 & 0 \\ -1 & 4 & -5 & 0 \\ 1 & -4 & -2 & 0 \end{bmatrix}$$

Row-reducing until the matrix is in REF (DO IT!), we get:

$$\begin{bmatrix} 1 & -4 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Fast way: There are 3 pivots in the coefficient matrix, and hence as many pivots as columns (in the coefficient matrix), and hence the vectors are linearly independent.

Slow way: Row-reducing further until we get the RREF:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

But this implies that $x = 0, y = 0, z = 0$, and hence $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Therefore, the equation $Ax = \mathbf{0}$ has only the trivial solution, and therefore the vectors are linearly independent.

Note: It's **VERY** important that you row-reduce until you get the REF. Otherwise you **CANNOT** conclude how many pivots (or free variables) there are. There's no way around this, any other way is considered incorrect!