MATH 54 – SOLUTION TO 10.2.11

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\begin{aligned}
A_0 =& \frac{\int_{-2}^{2} f(x) 1 dx}{\int_{-2}^{2} 1^2 dx} \\
=& \frac{1}{4}\left(\int_{-2}^{0} 1 dx + \int_{0}^{2} x dx\right) \\
=& \frac{1}{4}\left(2 + \left[\frac{x^2}{2}\right]_0^2\right) \\
=& \frac{1}{4}(2+2) \\
=& 1
\end{aligned}
$$

$$
\begin{aligned}
A_m =& \frac{\int_{-2}^{2} f(x)\cos\left(\frac{\pi m x}{2}\right) dx}{\int_{-2}^{2} \cos^2\left(\frac{\pi m x}{2}\right) dx} \\
=& \frac{1}{2}\left(\int_{-2}^{0} \cos\left(\frac{\pi m x}{2}\right) dx + \int_{0}^{2} x\cos\left(\frac{\pi m x}{2}\right) dx\right) \\
=& \frac{1}{2}\left(\left[\frac{2}{\pi m}\sin\left(\frac{\pi m x}{2}\right)\right]_{-2}^{0} + \left[\frac{2}{\pi m} x \sin\left(\frac{\pi m x}{2}\right)\right]_0^2 - \int_0^2 \frac{2}{\pi m}\sin\left(\frac{\pi m x}{2}\right) dx\right) \\
=& \frac{1}{2}\left(0 + 0 - \frac{2}{\pi m}\left[-\frac{2}{\pi m}\cos\left(\frac{\pi m x}{2}\right)\right]_0^2\right) \\
=& \frac{2}{(\pi m)^2}\left(\cos(\pi m) - \cos(0)\right) \\
=& \frac{2}{(\pi m)^2}\left((-1)^m - 1\right)
\end{aligned}
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$B_0 = 0$ by convention.

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\begin{aligned}
B_m =& \frac{\int_{-2}^{2} f(x) \sin\left(\frac{\pi m x}{2}\right) dx}{\int_{-2}^{2} \sin^2\left(\frac{\pi m x}{2}\right) dx} \\
=& \frac{1}{2}\left(\int_{-2}^{0} \sin\left(\frac{\pi m x}{2}\right) dx + \int_{0}^{2} x \sin\left(\frac{\pi m x}{2}\right) dx\right) \\
=& \frac{1}{2}\left(\left[-\frac{2}{\pi m}\cos\left(\frac{\pi m x}{2}\right)\right]_{-2}^{0} + \left[-\frac{2}{\pi m} x \cos\left(\frac{\pi m x}{2}\right)\right]_{0}^{2} + \int_{0}^{2} \frac{2}{\pi m}\cos\left(\frac{\pi m x}{2}\right) dx\right) \\
=& \frac{1}{2}\left(-\frac{2}{\pi m} + \frac{2}{\pi m}\cos(\pi m) - \frac{4}{\pi m}\cos(\pi m) + 0 + \left[\frac{4}{(\pi m)^2}\sin\left(\frac{\pi m x}{2}\right)\right]_{0}^{2}\right) \\
=& \frac{1}{2}\left(-\frac{2}{\pi m} + \frac{2}{\pi m}(-1)^m - \frac{4}{\pi m}(-1)^m + 0 - 0\right) \\
=& -\frac{1}{\pi m} - \frac{(-1)^m}{\pi m} \\
=& \frac{1}{\pi m}\left(-1 + (-1)^{m+1}\right)
\end{aligned}
$$

It follows that:

$$
f(x) \text{``} = \text{''} 1 + \sum_{m=1}^{\infty}\left(\frac{2}{(\pi m)^2}\left((-1)^m - 1\right)\cos\left(\frac{\pi m x}{2}\right) + \frac{1}{\pi m}\left(-1 + (-1)^{m+1}\right)\sin\left(\frac{\pi m x}{2}\right)\right)
$$