Here are a couple of hints to Homework 25. Enjoy! :)  

**Section 10.3: Fourier Series**

**10.3.1, 10.3.5.** $f$ is even if $f(-x) = f(x)$, $f$ is odd if $f(-x) = -f(x)$.

**10.3.7.** Just calculate $fg(-x) = f(-x)g(-x)$

For what follows, use the following formulas:

\[
    f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \left( \frac{n\pi x}{T} \right) + b_n \sin \left( \frac{n\pi x}{T} \right) \right\}
\]

\[
a_n = \frac{1}{T} \int_{-T}^{T} f(x) \cos \left( \frac{n\pi x}{T} \right) \, dx
\]

\[
b_n = \frac{1}{T} \int_{-T}^{T} f(x) \sin \left( \frac{n\pi x}{T} \right) \, dx
\]

Where $T$ is such that $f$ is defined on $(-T, T)$

**10.3.11.** To calculate the integral, split up the integral from $\int_{-2}^{0} + \int_{0}^{2}$

**10.3.17, 10.3.19.** The Fourier series converges to $f(x)$ if $f$ is **continuous** at $x$, and converges to $\frac{f(x^+) + f(x^-)}{2}$ if $f$ is **discontinuous** at $x$. As for the endpoints $T$ and $-T$, the fourier series converges to the average of $f$ at those endpoints.
10.3.26. Just show:

\[
\int_{-1}^{1} \cos \left( \frac{(2m-1)\pi}{2} x \right) \sin \left( \frac{(2n-1)\pi}{2} x \right) dx = 0
\]

\[
\int_{-1}^{1} \cos \left( \frac{(2m-1)\pi}{2} x \right) \cos \left( \frac{(2n-1)\pi}{2} x \right) dx = 0
\]

\[
\int_{-1}^{1} \sin \left( \frac{(2m-1)\pi}{2} x \right) \sin \left( \frac{(2n-1)\pi}{2} x \right) dx = 0
\]

for all \( m \) and \( n \).

Use the following formulas:

\[2 \cos(A) \cos(B) = \cos(A+B) + \cos(A-B),\]

\[2 \sin(A) \sin(B) = \cos(A-B) - \cos(A+B)\]

as well as the fact that odd-ness (for the first one).

10.3.27. Just calculate:

\[
\frac{\int_{-1}^{1} f(x)g(x)dx}{\int_{-1}^{1} g(x)^2dx}
\]

for every function \( g(x) \) in 10.3.27 (this follows from formula (20) on page 588).