MATH 54 – HINTS TO HOMEWORK 25

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Here are a couple of hints to Homework 25. Enjoy! :)

SECTION 10.3: FOURIER SERIES

10.3.1, 10.3.5. f is even if $f(-x) = f(x)$, f is odd if $f(-x) = -f(x)$.

10.3.7. Just calculate $fg(-x) = f(-x)g(-x)$

For what follows, use the following formulas:

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{T}\right) + b_n \sin\left(\frac{n\pi x}{T}\right) \right\}$$

$$a_n = \frac{1}{T} \int_{-T}^{T} f(x) \cos\left(\frac{n\pi x}{T}\right) dx$$

$$b_n = \frac{1}{T} \int_{-T}^{T} f(x) \sin\left(\frac{n\pi x}{T}\right) dx$$

Where T is such that f is defined on $(-T, T)$

10.3.11. To calculate the integral, split up the integral from $\int_{-2}^{0} + \int_{0}^{2}$

10.3.17, 10.3.19. The Fourier series converges to $f(x)$ if f is **continuous** at x, and converges to $\frac{f(x^+)+f(x^-)}{2}$ if f is **discontinuous** at x. As for the endpoints T and $-T$, the fourier series converges to the average of f at those endpoints.

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10.3.26. Just show:

$$\int_{-1}^{1} \cos\left(\frac{(2m-1)\pi}{2}x\right)\sin\left(\frac{(2n-1)\pi}{2}x\right)dx = 0$$
$$\int_{-1}^{1} \cos\left(\frac{(2m-1)\pi}{2}x\right)\cos\left(\frac{(2n-1)\pi}{2}x\right)dx = 0$$
$$\int_{-1}^{1} \sin\left(\frac{(2m-1)\pi}{2}x\right)\sin\left(\frac{(2n-1)\pi}{2}x\right)dx = 0$$

for all m and n.

Use the following formulas:

$$2\cos(A)\cos(B) = \cos(A+B)+\cos(A-B), 2\sin(A)\sin(B) = \cos(A-B)-\cos(A+B)$$

as well as the fact that odd-ness (for the first one).

10.3.27. Just calculate:

$$\frac{\int_{-1}^{1} f(x)g(x)dx}{\int_{-1}^{1} g(x)^2dx}$$

for every function $g(x)$ in $10.3.27$ (this follows from formula (20) on page 588).