MATH 54 – SOLUTION TO 9.5.31

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Eigenvalues:

$$\det(\lambda I - A) = (\lambda - 1)^2 - 9 = 0 \Rightarrow \lambda - 1 = \pm 3 \Rightarrow \lambda = -2, 4$$

Eigenvectors:

$$Nul(-2I - A) = Nul \begin{bmatrix} -3 & -3 \\ -3 & -3 \end{bmatrix} = Nul \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = Span \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

$$Nul(4I - A) = Nul \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = Nul \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = Span \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

Therefore, the general solution to our differential equation is:

$$\mathbf{x}(t) = Ae^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + Be^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now plug in $t = 0$:

$$\mathbf{x}(0) = A \begin{bmatrix} 1 \\ -1 \end{bmatrix} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

which leads us to solve the system:

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Row-reducing, we get:

$$\begin{bmatrix} 1 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

which gives us $A = 1$, $B = 2$, and therefore our solution is:

$$\mathbf{x}(t) = e^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 2e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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