

MATH 54 – SOLUTION TO 9.4.16

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The pre-Wronskian is:

$$\widetilde{W}(t) = \begin{bmatrix} \sin(t) & \sin(2t) \\ \cos(t) & \cos(2t) \end{bmatrix}$$

Evaluating this at $t = \frac{\pi}{2}$ (notice that 0 or π don't work), we get:

$$\widetilde{W}\left(\frac{\pi}{2}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

And therefore the Wronskian at $\frac{\pi}{2}$ is:

$$W\left(\frac{\pi}{2}\right) = \det \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = -1$$

Since the Wronskian is nonzero at some point (here $\frac{\pi}{2}$), the two vector functions are linearly independent on \mathbb{R} .