MATH 54 – HINTS TO HOMEWORK 22

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Here are a couple of hints to Homework 22. Enjoy!

SECTION 9.1: INTRODUCTION

9.1.7. Let $z = y'$, then $z' = y'' = -\frac{b}{m}y' - \frac{k}{m}y = -\frac{b}{m}z - \frac{k}{m}y$, then we get:

$$\begin{cases} y' = z \\ z' = -\frac{k}{m}y - \frac{b}{m}z \end{cases}$$

9.1.10. Similar to **9.1.7**

9.1.13. Let

$$\begin{cases} x_1 = x \\ x_2 = x' = x_1' \\ x_2' = x'' = 3x' - t^2 y + \cos(t)x \\ x_3 = y \\ x_4 = y' = x_3' \\ x_5 = y'' = x_4' \\ x_5' = y''' = -y'' + tx' - y' - e^t x \end{cases}$$

Now calculate x_i' and put them in a system, using the above relations.

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SECTION 9.4: LINEAR SYSTEMS IN NORMAL FORM

9.4.3. This problem is easier to do than to explain.

For example, for $9.4.1$:

$$A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} t^2 \\ e^t \end{bmatrix}$$

Just beware of the following: If for example $y'(t)$ doesn't contain $x(t)$, then the corresponding term in the matrix A is 0. Be inspired by this to solve $9.4.3$.

9.4.7. The trick is to let $x = w'$, $y = w''$, $z = w'''$, then:

$$\begin{cases} w' = x \\ x' = y \\ y' = z \\ z' = -w + t^2 \end{cases}$$

Now rewrite this system in matrix form.

9.4.13, 9.4.16, 9.4.19. Use the Wronskian! The good news is that the wronskian is very easy to calculate! Just ignore any constants and put all the two or three vectors in a matrix. For example, for $9.4.17$, the (pre)-Wronskian is:

$$\widetilde{W}(t) = \begin{bmatrix} e^{2t} & e^{2t} & 0 \\ 0 & e^{2t} & e^{3t} \\ 5e^{2t} & -e^{2t} & 0 \end{bmatrix}$$

And as usual, pick your favorite point t_0, and evaluate $det(\widetilde{W}(t_0))$. If this is nonzero, your functions are linearly independent.

9.4.23. Just show that the three vectors are linearly independent. To find A, for every vector $\mathbf{x}$ given, calculate $\mathbf{x}'(t)$ for every vector $\mathbf{x}$ and just write this in terms of $\mathbf{x}(t)$. This gives the first, second, and third column of A respectively.

9.4.27. Linear operator (in this case) is just another word for linear transformation. Just show that $L[\mathbf{x} + \mathbf{y}] = L[\mathbf{x}] + L[\mathbf{y}]$ and that $L[c\mathbf{x}] = cL[\mathbf{x}]$