MATH 54 – HINTS TO HOMEWORK 21

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Here are a couple of hints to Homework 21. Enjoy!

SECTION 6.1: BASIC THEORY OF LINEAR DIFFERENTIAL EQUATIONS

6.1.3. First of all, make sure that the coefficient of y''' is equal to 1. Then look at the domain of each term, including the inhomogeneous term (more precisely, the part of the domain which contains the initial condition 5). Then the answer is just the intersection of the domains you found!

6.1.7. Use the Wronskian with $x = 0$

6.1.17. Verify that the three functions solve the differential equations, then show they're linearly independent (by using the Wronskian at $x = 1$)

6.1.19. Use the fact that $y = y_p + y_0$, where y_p is the given particular solution, and y_0 is the general solution to the homogeneous equation (which is the span of the fundamental solution set). Then use the initial conditions to solve for the constants.

6.1.23. For example, for (a), we have:

$$L[2y_1 - y_2] = 2L[y_1] - L[y_2] = 2x\sin(x) - (x^2 + 1) = 2x\sin(x) - x^2 - 1$$

So $2y_1 - y_2$ solves the equation for (a)

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6.1.27. Either you can use the Wronskian with $x = 0$, or use the following reasoning: If

$$a_0 + a_1 x + a_2 x^2 \cdots + a_n x^n = 0$$

This means that for **EVERY** x, x is a zero of $a_0 + a_1 x + a_2 x^2 \cdots + a_n x^n$ (by definition of the zero function). However, this polynomial is of degree n, hence cannot have more than n zeros unless $a_1 = a_2 = \cdots = a_n = 0$, which we want!

Section 6.2: Homogeneous linear equations with constant coefficients

6.2.3, 6.2.7. The following fact might be useful:

Rational roots theorem: If a polynomial p has a zero of the form $r = \frac{a}{b}$, then a divides the constant term of p and b divides the leading coefficient of p.

This helps you 'guess' a zero of p. Then use long division to factor out p.

6.2.15. The reason this is written out in such a weird way is because the auxiliary polynomial is easy to figure out! Here, the auxiliary polynomial is

$(r-1)^2(r+3)(r^2+2r+5)^2$.

6.2.25. Suppose:

$$a_0 e^{rx} + a_1 x e^{rx} + \cdots + a_{m-1} x^{m-1} e^{rx} = 0$$

Now cancel out the e^{rx}, and you get:

$$a_0 + a_1 x + \cdots + a_{m-1} x^{m-1} = 0$$

But $1, x, x^2 \cdots, x^{m-1}$ are linearly independent, so $a_0 = a_1 = \cdots a_{m-1} = 0$, which is what we wanted!