MATH 54 – SOLUTION TO 4.3.21

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The auxiliary equation is $r^2 + 2r + 2 = 0$, which gives:

$$r = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

Which means that the general solution to the differential equation is:

$$y(t) = Ae^{-t}\cos(t) + Be^{-t}\sin(t)$$

Plugging in $t = 0$, we get $y(0) = A = 2$, so $\boxed{A = 2}$ and hence:

$$y(t) = 2e^{-t}\cos(t) + Be^{-t}\sin(t)$$

Differentiating, we get:

$$\begin{aligned} y'(t) =& -2e^{-t}\cos(t) + 2e^{-t}(-\sin(t)) - Be^{-t}\sin(t) + Be^{-t}\cos(t) \\ =&(B-2)e^{-t}\cos(t) + (-2-B)e^{-t}\sin(t) \end{aligned}$$

Plugging in $t = 0$, we get:

$y'(0) = (B - 2) = 1$, and so $\boxed{B = 3}$.

Therefore our solution is:

$$y(t) = 2e^{-t}\cos(t) + 3e^{-t}\sin(t)$$

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