Here are a couple of hints to Homework 19. Enjoy!

**SECTION 4.2: HOMOGENEOUS LINEAR EQUATIONS: THE GENERAL SOLUTION**

4.2.27. Linearly dependent, because $\sin(2t) = 2\cos(t)\sin(t)$

4.2.34.

(a) Just evaluate the determinant

(b) $(\Rightarrow)$ If there is some point $\tau$ where $W = 0$ at $\tau$, then by Lemma 1, $y_1$ and $y_2$ are linearly dependent.

$(\Leftarrow)$ Suppose that $ay_1(t) + by_2(t) = 0$ for all $t$. Then differentiating this, we get $ay_1'(t) + by_2'(t) = 0$, but then we have:

$$
\begin{bmatrix}
y_1(t) & y_2(t) \\
y_1'(t) & y_2'(t)
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

But since $W$ is never 0 on $I$, the determinant of the first matrix is nonzero, and hence that matrix is invertible, and hence $a = 0$ and $b = 0$, so $y_1$ and $y_2$ are linearly independent on $I$.

(c) First assume that $y_1 = cy_2$, and calculate $W[y_1, y_2] = W[cy_2, y_2]$ and show it’s equal to 0. Then assume that $y_2 = cy_1$ and calculate $W[y_1, y_2] = W[y_1, cy_1]$ and show that you get 0 in both cases.

**SECTION 4.3: AUXILIARY EQUATIONS WITH COMPLEX ROOTS**

The problems should hopefully be pretty straightforward :)

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*Date: Wednesday, November 12th, 2014.*
4.3.29(b). The following fact might be useful:

**Rational roots theorem:** If a polynomial $p$ has a zero of the form $r = \frac{a}{b}$, then $a$ divides the constant term of $p$ and $b$ divides the leading coefficient of $p$.

This helps you ‘guess’ a zero of $p$. Then use long division to factor out $p$. 