MATH 54 – HINTS TO HOMEWORK 19

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Here are a couple of hints to Homework 19. Enjoy!

SECTION 4.2: HOMOGENEOUS LINEAR EQUATIONS: THE GENERAL SOLUTION

4.2.27. Linearly dependent, because $\sin(2t) = 2\cos(t)\sin(t)$

4.2.34.

(a) Just evaluate the determinant

(b) ($\Rightarrow$) If there is some point τ where $W = 0$ at τ, then by Lemma 1, y_1 and y_2 are linearly dependent.
($\Leftarrow$) Suppose that $ay_1(t) + by_2(t) = 0$ for all t. Then differentiating this, we get $ay_1'(t) + by_2'(t) = 0$, but then we have:

$$\begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

But since W is never 0 on I, the determinant of the first matrix is nonzero, and hence that matrix is invertible, and hence $a = 0$ and $b = 0$, so y_1 and y_2 are linearly independent on I.

(c) First assume that $y_1 = cy_2$, and calculate $W[y_1, y_2] = W[cy_2, y_2]$ and show it's equal to 0. Then assume that $y_2 = cy_1$ and calculate $W[y_1, y_2] = W[y_1, cy_1]$ and show that you get 0 in both cases.

SECTION 4.3: AUXILIARY EQUATIONS WITH COMPLEX ROOTS

The problems should hopefully be pretty straightforward :)

Date: Wednesday, November 12th, 2014.

4.3.29(b). The following fact might be useful:

Rational roots theorem: If a polynomial p has a zero of the form $r = \frac{a}{b}$, then a divides the constant term of p and b divides the leading coefficient of p.

This helps you 'guess' a zero of p. Then use long division to factor out p.