

MATH 54 – SOLUTION TO 6.7.16

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Notice that:

$$\begin{aligned}\|\mathbf{u} - \mathbf{v}\|^2 &= (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) \\ &= \mathbf{u} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} \quad (\text{since } \mathbf{u} \text{ and } \mathbf{v} \text{ are orthogonal}) \\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 \\ &= 1 + 1 \quad (\text{by orthonormality}) \\ &= 2\end{aligned}$$

Therefore $\|\mathbf{u} - \mathbf{v}\|^2 = 2$, and hence $\|\mathbf{u} - \mathbf{v}\| = \sqrt{2}$ (by taking square roots)