

MATH 54 – HINTS TO HOMEWORK 18

PEYAM TABRIZIAN

Here are a couple of hints to Homework 18. Enjoy!

SECTION 6.7: INNER PRODUCT SPACES

6.7.1. Here $\langle \mathbf{x}, \mathbf{y} \rangle = 4u_1v_1 + 5u_2v_2$

6.7.5, 6.7.7. Here $\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$. And $\|p\| = \sqrt{\langle p, p \rangle}$. Finally, remember that the formula for orthogonal projection remains the same, namely:

$$\hat{q} = \frac{\langle q, p \rangle}{\langle p, p \rangle} p$$

6.7.11. Here $\langle p, q \rangle = p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1) + p(2)q(2)$.

If we let $p_3 = t^2$, then we have:

$$\hat{p}_3 = \frac{\langle p_3, p_0 \rangle}{\langle p_0, p_0 \rangle} p_0 + \frac{\langle p_3, p_1 \rangle}{\langle p_1, p_1 \rangle} p_1 + \frac{\langle p_3, p_2 \rangle}{\langle p_2, p_2 \rangle} p_2$$

6.7.16. This is very cute! Calculate $\|\mathbf{u} - \mathbf{v}\|^2$ in terms of dot products, and show that the result is 2. Make sure to justify every step! Also, what property allows you to conclude that the answer is $\sqrt{2}$, and not $-\sqrt{2}$?

SECTION 7.1: DIAGONALIZATION OF SYMMETRIC MATRICES

7.1.9. Remember orthogonal matrices have **orthonormal** columns!

7.1.17. First diagonalize the matrix as usual, and then apply Gram-Schmidt to each eigenspace!

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7.1.25.

- (a) T (by theorem 2; this is the most important fact about symmetric matrices!)
- (b) T (by theorem 1)
- (c) F (take the identity matrix for example)
- (d) F (\mathbf{v} has to be a *unit* vector)