MATH 54 – SOLUTION TO 6.5.11

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(a) Denote the columns of A by $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$

$$\begin{aligned}
\hat{\mathbf{b}} &= \left(\frac{\mathbf{b} \cdot \mathbf{a_1}}{\mathbf{a_1} \cdot \mathbf{a_1}}\right)\mathbf{a_1} + \left(\frac{\mathbf{b} \cdot \mathbf{a_2}}{\mathbf{a_2} \cdot \mathbf{a_2}}\right)\mathbf{a_2} + \left(\frac{\mathbf{b} \cdot \mathbf{a_3}}{\mathbf{a_3} \cdot \mathbf{a_3}}\right)\mathbf{a_3} \\
&= \frac{36}{54}\begin{bmatrix} 4 \\ 1 \\ 6 \\ 1 \end{bmatrix} + \frac{0}{27}\begin{bmatrix} 0 \\ -5 \\ 1 \\ -1 \end{bmatrix} + \frac{9}{27}\begin{bmatrix} 1 \\ 1 \\ 0 \\ -5 \end{bmatrix} \\
&= \frac{2}{3}\begin{bmatrix} 4 \\ 1 \\ 6 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 0 \\ -5 \\ 1 \\ -1 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 0 \\ -5 \end{bmatrix} \\
&= \begin{bmatrix} 3 \\ 1 \\ 4 \\ -1 \end{bmatrix}
\end{aligned}$$

(b) Standard-way:

Solve $A\widetilde{\mathbf{x}} = \hat{\mathbf{b}}$ by row-reduction::

$$\begin{bmatrix} 4 & 0 & 1 & 3 \\ 1 & -5 & 1 & 1 \\ 6 & 1 & 0 & 4 \\ 1 & -1 & -5 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore:

$$\widetilde{\mathbf{x}} = \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{1}{3} \end{bmatrix}$$

Date: Wednesday, November 5, 2014.

OMG-way: Notice that in (a), we obtained:

$$\frac{2}{3}\begin{bmatrix}4\\1\\6\\1\end{bmatrix}+0\begin{bmatrix}0\\-5\\1\\-1\end{bmatrix}+\frac{1}{3}\begin{bmatrix}1\\1\\0\\-5\end{bmatrix}=\begin{bmatrix}3\\1\\4\\-1\end{bmatrix}$$

which tells us directly that:

$$A\begin{bmatrix}\frac{2}{3}\\0\\\frac{1}{3}\end{bmatrix}=\hat{\mathbf{b}}$$

therefore the least-squares solution is:

$$\widetilde{\mathbf{x}}=\begin{bmatrix}\frac{2}{3}\\0\\\frac{1}{3}\end{bmatrix}$$